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AC Machines I
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AC Electrical Machines

1-1 Introduction:

3-phase induction motors are simple, rugged, low cost, and easy to maintain. They run at essentially constant speed from zero-to-full load. Therefore, they are the motors most frequently encountered in industry

AC machines can be classified into two types:

Synchronous machines: - Alternators –Motor

A) Salient-Pole B) Cylindrical Rotors

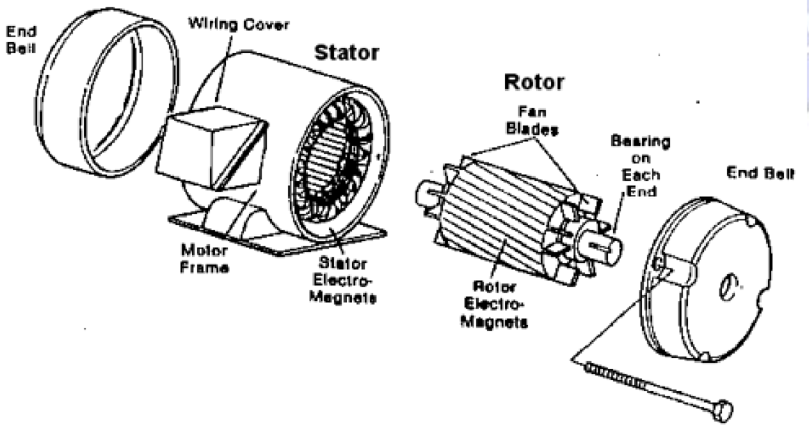
Asynchronous (induction) machines:- induction motors – induction generator

A) Squirrel –Cage B) Slip-Ring

1-2 Constructional Parts: -

An AC induction motor has two main parts:

1. Stator: consisting of a steel frame that supports a hollow, cylindrical core of stacked laminations. Slots on the internal circumference of the stator house the stator winding.
2. Rotor: also composed of punched laminations, with rotor slots for the rotor winding.
3. Shaft.
4. Bearing.
5. Yoke.



1-3 Materials: -

A) Electrical –Conductors (Winding)

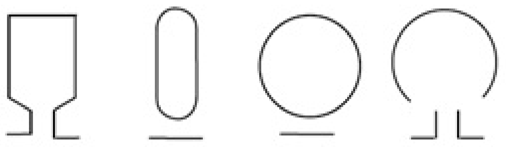
B) Magnetic – Stator and Rotor Cores (M. Circuit)

Note: - The air-gap between stator and rotor of a 3phase induction motor ranges from

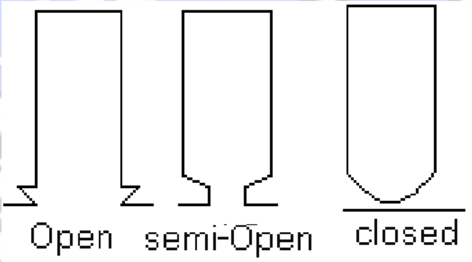
0.4-4mm

1-4 Types of Motor Slots:

Open, Closed, Semi Open, Semi Closed



Types of rotor slots

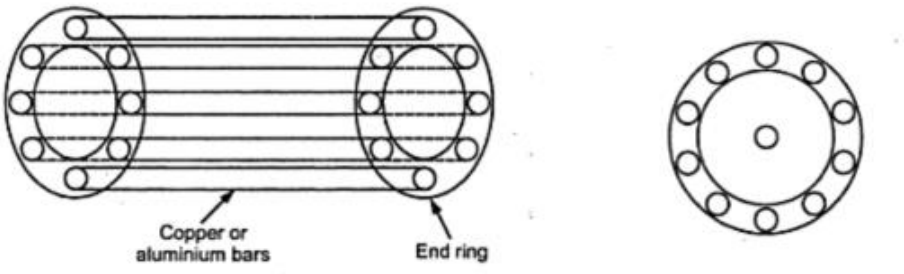


1-5 Types of Rotor Winding:

There are two-types of rotor windings:

1- **Squirrel-cage windings**, which produce a squirrel-*cage induction motor (most common)*

Squirrel cage rotor consists of copper bars, slightly longer than the rotor, which are pushed into the slots. The ends are welded to copper end rings, so that all the bars are short circuited. In small motors, the bars and end-rings are die cast in aluminum to form an integral block.



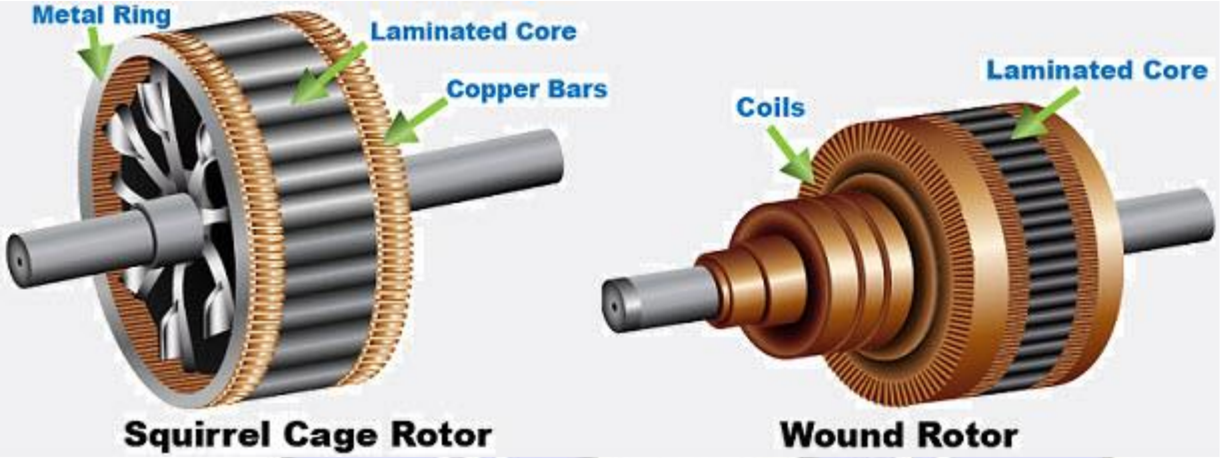
(a) Cage type structure

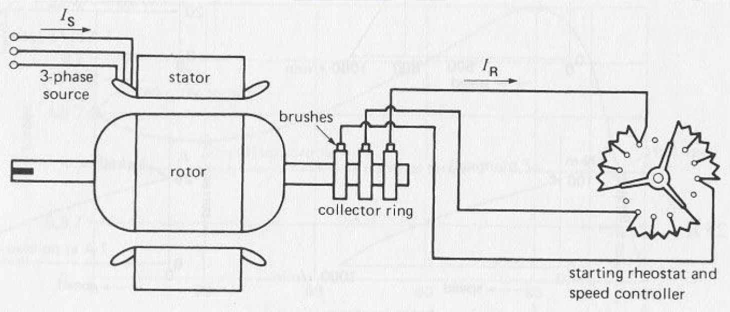
(b) Symbolic representation

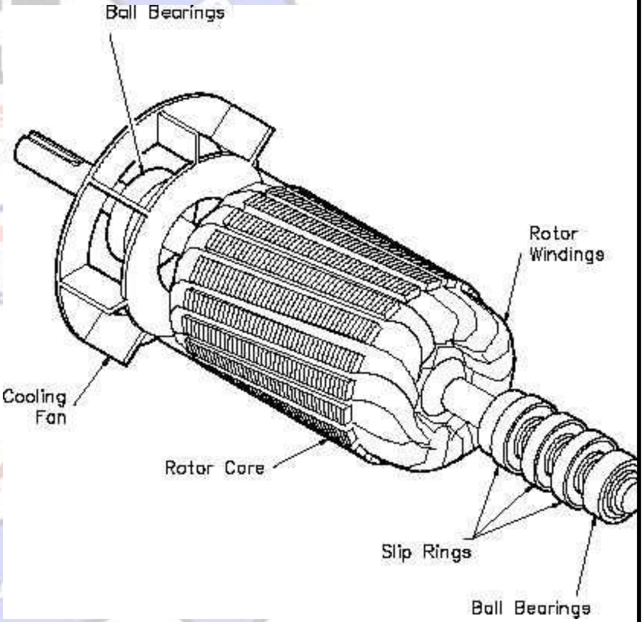
2- Conventional 3-phase windings made of insulated wire, which produce a ***Wound-Rotor Induction Motor or Slip Ring Induction Motor (special characteristics)***

A wound rotor has a 3-phase winding, similar to the stator winding.

The rotor winding terminals are connected to three slip rings which turn with the rotor. The slip rings/brushes allow external resistors to be connected in series with the winding. The external resistors are mainly used during start-up – under normal running conditions the windings short circuited externally.







<u>1-6 Winding</u>: -

Choice of winding

1- Type of Coil: Concentric, Lap, Wave

2- Overhang: - diamond, multi plane ,mash

3-Layers: - single-layer, double –layer

4-Slots: - open-closed, semi closed

5-Connections: Star, Delta

6-Phase Spread: - 60°, 120°

7-Slotting: - Integral, Fractional

8-Coil-Span:- Full-Pitch, Chorded

9-Circuits: - Series, Parallel

10- Coils: - Single-Turn, Multi Turn

1-7 Symbols:

S-total no. of slots in the stator

C-total no. of coils

P- no. of pole pairs

$\propto -slot\ pitch(elec.radious)$

$\boldsymbol{Q} - no.of\ slots/polepair$

q -*no .of slots/pole/phase*

б $- spread\ of\ phase - group$

a - *no. of parallel circuits/phase wending*

m - *no.of phase,*   ***y*** - *coil spa***n**

1-8 main equation

$$Q=S/P$$

$$q= Q/2m= S/2pm$$

$$\alpha = \frac{2\pi p}{s} = \frac{2\pi}{Q}$$

$$\sigma = q\alpha = \frac{\pi}{m}$$

$$\mathcal{T} = \frac{s}{2p} = \frac{Q}{2} = qm \quad pole - pitch$$

If y=𝒯→ full pitch

Ns=conductors/slot

Z=total conductors in machine

Ts=total turns connected in series/phase

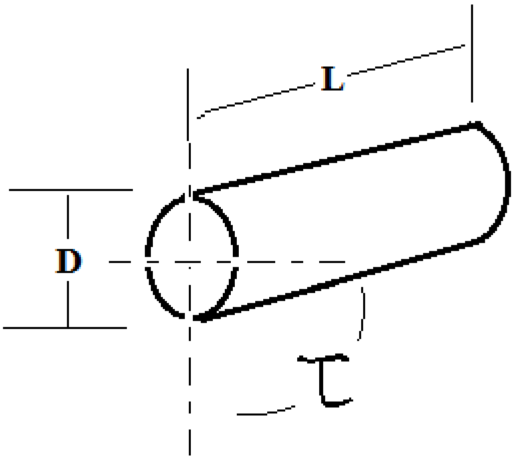
Tc=turns/coil

Tc=1 single-turn coil

Tc>1 multi-turn coil

Ns=Z/S ,Z=S Ns ,c=Z/2Tc

Ts=Z/2m= SNs/2m

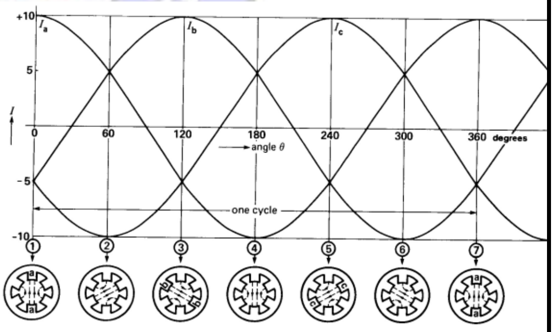


1-9 Induced EMF

F=P.n (R/S) =P.n/60 (R/M)

n =NO. of Turns per min.

e =E_m sin (wt), E_m=maximum E.M.F



E_m= B L v v= peripheral speed

$$v = (\pi D)n = (2p\tau)n = 2f\tau \ \ ,$$

$$D = dimeter$$

$$Bav = \frac{\emptyset}{\tau L} = \frac{2P\emptyset}{\pi \mathrm{DL}}$$

$$Em = \frac{2P\emptyset}{\pi D\ L} \times L \times \pi D n = 2P\emptyset n$$

$$Erms = Kf\ Em$$

$$kf = form\ factor = 1.11 = \frac{\pi}{2\sqrt{2}}$$

$$Erms = 2.22\ P\ \emptyset \frac{F}{P} = 2.22\emptyset\ F$$

1-10 Skewing effect

Rotor conductors are skewed because of these main reasons:

Primarily to prevent the cogging phenomenon. It is a phenomenon in which, if the rotor conductors are straight, there are chances of magnetic locking or strong coupling between rotor & stator.

To avoid crawling. Crawling is a phenomenon where harmonic components introduces oscillations in torque

Other reasons include increasing effective rotor resistance, to improve the starting torque & starting power factor, increasing effective magnetic coupling between stator & rotor fluxes.

$$ks = \sin\frac{\gamma}{2} / \frac{\gamma}{2}$$

ɤ=skewing angle

$$Es = Erms = 2.22\ F\ Ks\ \emptyset$$

Harmonic Effect

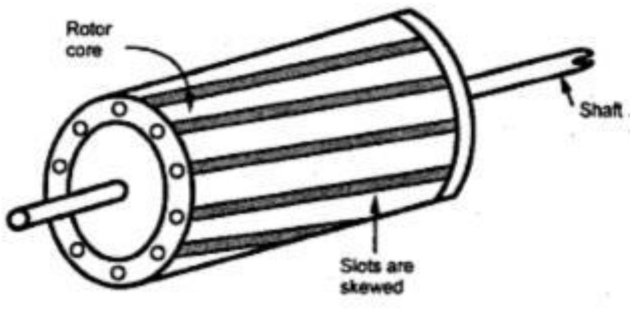
N=harmonic order=1,2,3,4, ……∞

N=2km±1=1,2,3,4   k=0,1,2,……..

N=4km± 1=1,3,5,7….   Wave positive &negative area eque

N =6k ±1 =1,5,7,11,13,…..3Ø machines

M, no.of phase



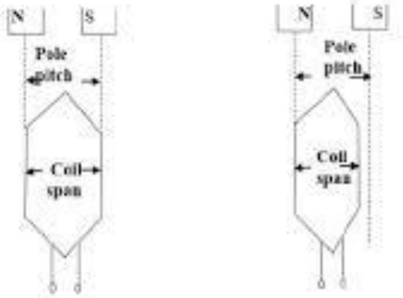
$$En = E1\,\frac{Bn}{B1}\,emf\ induced\ harmonic\ field$$

$$E = (E1^2 + E3^2 + \cdots \ldots .En^2)½ \quad resultant\ \ induced\ emf$$

1-11 Pitch factor: is the ratio of the voltage induced in a short-pitch winding to the voltage that would be induced if the winding were full pitch

In short pitched coil, the induced emf of two coil sides get vectorially added and give resultant emf of the loop. In short pitched coil, the phase angle between the induced emf of two opposite coil sides is less than 180° (electrical). But we know that, in full pitched coil, the phase angle between the induced emf of two coil sides is exactly 180° (electrical). Hence, the resultant emf of a full pitched coil is just the arithmetic sum of the emfs induced on both sides of the loop. We well know that vector sum or phasor sum of two quantities is always less than their arithmetic sum. The pitch factor is the measure of resultant emf of a short-pitched coil in comparison with resultant emf of a full pitched coil.

$$K_p = \frac{Resultant\ emf\ of\ short\ pitch\ coil}{Resultant\ emf\ of\ full\ pitch\ coil}$$



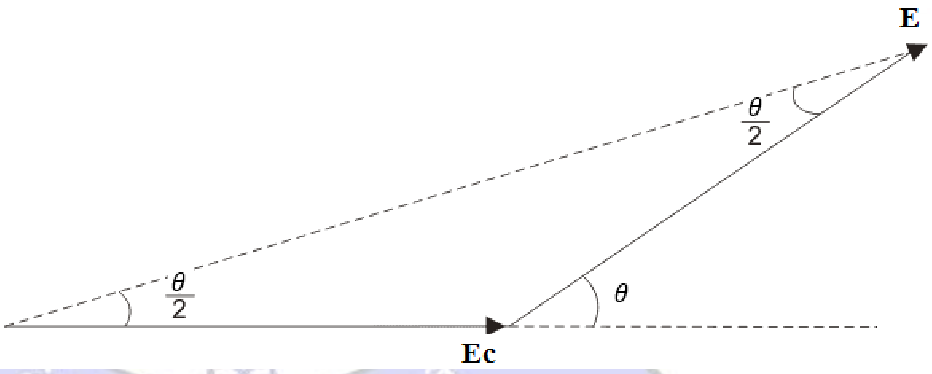
e_c emf induced in turn side.

e. total emf induced in turn

$e=2e_c$ for full.pitch turn

$$\sin\frac{\Upsilon}{2} = \frac{ad}{ab} = \frac{\frac{e}{2}}{e1 \text{ or } e2}$$

$$or\ E = 2Ec\ \sin\frac{\Upsilon}{2}$$



$$\mathrm{E} = 2\mathrm{Kp\ Ec} \qquad \mathrm{Kp} = \sin\frac{\Upsilon}{2} or\ \ Kp = \cos\frac{\theta}{2} \quad in\ Similar\ way$$

$$Et = emf\ \ induced\ \ in\ single\ \ turn$$

$$Et = 4.44\ F\ Kp\ \emptyset$$

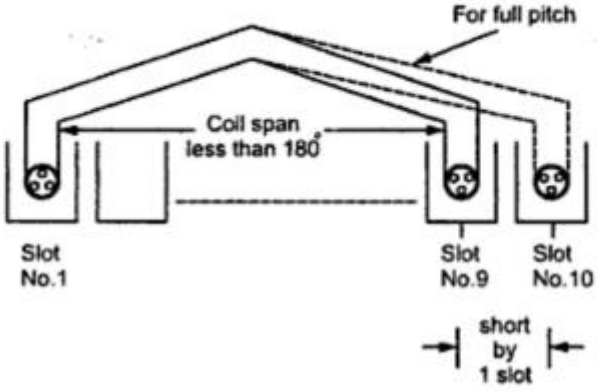
$$Kp = 1\ FOR\ Full\ pitch\ windings$$

$$Kp < 1\ FOR\ Chorded\ winding$$

$$\theta = y\ \alpha\ \ , \qquad \Upsilon = 180 - \theta$$

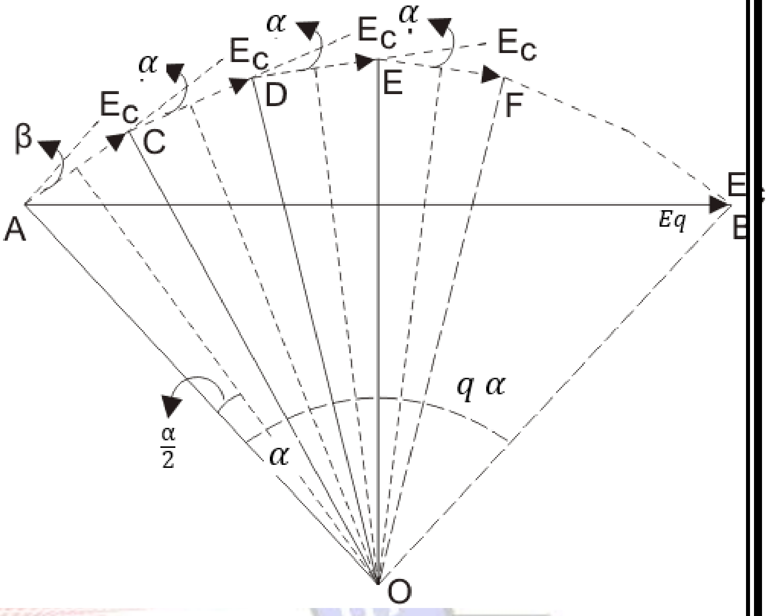
$$Ec = emf\ induced\ in\ acoil\ with\ Tc\ turns$$

$$Ec = 4.44\ F\ KP\ Tc\ \emptyset$$



$$KPn = \sin\frac{n\gamma}{2} = \cos\frac{n\theta}{2}$$

1-12 Distribution Factor:



$$k = \frac{Phasor\ sum\ of\ component\ emfs}{Arithmetic\ sum\ of\ component\ emfs}$$

$$q\ Ec = 3Ec = E1 + E2 + E3$$

$$Kd = \frac{AB}{q\ Ec} = \frac{Eq}{q\ Ec}$$

$$AB = Eq = Kd(q\ Ec)$$

$$= 2R\ \sin\frac{q\ \alpha}{2} = \left(\frac{\sin\frac{q\alpha}{2}}{q\sin\frac{\alpha}{2}}\right)(q\ Ec)$$

$$R = \frac{ab}{2\sin\frac{\alpha}{2}} = \frac{Ec}{2\sin\frac{\alpha}{2}}$$

$$\therefore kd = \frac{\sin\frac{q\alpha}{2}}{q\sin\frac{\alpha}{2}} \rightarrow kdn = \frac{\sin\frac{qn\alpha}{2}}{q\sin\frac{n\alpha'}{2}}$$

$$kw = kp\ kd$$

$$Eq = 4.44\ F\ Kw\ (q\ Tc)\emptyset\ , \qquad Ts = pq\ Tc$$

$$Eph = 4.44\ F\ Kw\ Ts\ \emptyset$$

$$EL = \sqrt{3}Eph \quad for\ \lambda\ connected\ winding$$

$$EL = Eph\ \ for\ \Delta connected\ winding$$

1-13 Produced m.m.f

$$i = IM\ \sin wt$$

$$Fc = iT = \sqrt{2}I\ Tc\ \sin wb$$

$$F = \frac{Fc}{2} = \pm\frac{iTc}{2} = \pm\frac{\sqrt{2}ITc}{2}\sin \mathrm{wt} = \pm \mathrm{Fm}\ \sin \mathrm{wt}$$

$$Fm = \frac{\sqrt{2}I\ Tc}{2}; F1 = \frac{4}{\pi}Fm$$

$$\boldsymbol{or}\ F1 = \frac{2\sqrt{2}}{\pi}I\ Tc = 0.9ITc$$

$$Fn = {}^{F1}/_{n} = 0.9\frac{I\ Tc}{\mathrm{n}}$$

For distributed coils(q=2)

f1m=f2m=fm

Kd=fqm/qfm=f1/qfm

F1=qkdfm=0.9i(q Tc)kd   single layer

F2= qkwfm=1.8i(q Tc)kw   double layer

I=Iph/a ;Ts=pqTs

Single-layer Ts=pqTc/a

Double-layer Ts=2pq Tc/a

IphTs=aI 2pq Tc/a=2p(IqTc)

Fn=0.9 IphTs Kw/n p   for single phase

For 3Ø f1=m/2f1=3/2f1=1.35 iphTs kw/p

(fn)3Ø=1.35 IphTs kw/np

In general

(fn)m=m/2 fn=0.45 m iphkwnTs/np

*for uniform syn. System (f) is constant and rotating with syn. speed.

*if the reluctance constant ,the flux and mmf form is the same the mmf direction of rotation depends on the phase sequence.

*slot skewing has no effect of mmf value.

*mmf wave form is sinusoidal if s=∞(lowest harmonic)

*if the field under both poles is the same,then the horizontal axis will divide the mmf waveform in two equal areas.

*in poly phase machines ,the mmf resultant phase or (f) is varying in time and rotating is same.

*if the phasor value is always constant ,then the rotating field is circular otherwise if is ellipsoid for single phase it is pulsating

Ex. 1:- a 6-pole,generator having (108) slots on the stator find the pitch factor for the fundamented,1st harmonic when the coil is chorded by(3)slots.

Sol:-

$s = 108 \ \ , 2P = 6 \ , \quad P = 3 \ \ , y = 3$

$Q = \frac{S}{P} = \frac{108}{3} = 36 \ \ , \ \ \propto = \frac{2\pi}{Q} = \frac{360}{36} = \mathring{10} \ , \theta = y\alpha = 3 \times \mathring{10} = 30 - Charding\ angle$

$\gamma = 180 - \theta = 180 - 30 = 150° \ coil\ span\ angle$

$kp1 = \sin\frac{\gamma}{2} = \sin 75° = 0.962$

$or\ kp1 = \cos\frac{\theta}{2} = \cos 15 = 0.962$

Ex.2) a 3-phase ,16pole,Y-connected generator ,its air gap flux is 60 mwb/pole sinusoidally the stator having(96)slots in each(4)conductors, and the coil span is(150).find the line emf induced at no load, when the generator is running at 375rpm

Sol:- $\gamma = 150° \ , s = 96 \ , \ 2P = 16 \ \ , \emptyset = 60 \times \frac{10^{3} wb}{pole} \ \ , Ns = 4 \ , n = 375rpm\ F = \frac{nP}{60} = \frac{375\times 8}{60} = 50HZ \ \ , q = \frac{s}{2Pm} = \frac{96}{16\times 3} = 2$

$Ts = \frac{sNs}{2m} = \frac{96\times 4}{2\times 3} = 64 \ , \propto = \frac{60°}{2} = 30° \ , kp = \sin\frac{150}{2} = 0.966 \ , \ kd = \frac{\sin\frac{q\propto}{2}}{q\sin\frac{\propto}{2}} = \frac{\sin 30°}{2\sin 15°} = 0.962$

$E = 4.44Fkp\ kd\ Ts\ \emptyset = 4.44 \times 50 \times 0.966 \times 0.962 \times 64 \times 0.06 = 792.2V$

$E = \sqrt{3E} = \sqrt{3} \times 792.2 = 1375\ v$

Ex. 3) a 3-phase, 50Hz generator running at 1500 rpm, the stator inside diameter is (0.5m) and its length is (1.2m)find the induced emf in each conductor. if the maximum air gap flux density is(0.62T) and it is sinusoidally distributed.

Sol:-

$$F = 50 \quad , n = 1500rpm \, , D = 0.5m \quad , L = 1.2m \quad Bm = 0.62T$$

$$P = \frac{60F}{n} = \frac{60 \times 50}{1500} = 2 \qquad 2P = 4$$

$$\tau = \frac{\pi D}{2P} = \frac{\pi \times 0.5}{4} = 0.3925M \ \rightarrow Pole\ pitch$$

$$Bav = \frac{2}{\pi} Bm = \frac{2}{\pi} \times 0.62 = 0.392T \quad \rightarrow avarage\ flux\ density$$

$$\emptyset = Bav(\tau L) = 0.392 \times 0.3925 \times 1.2 = 0.187wb$$

Then emf induced in one conductor is:-

$$Econ = 2.22F\emptyset = 2.22 \times 50 \times 0.187 = 20.7v$$

Emf induced in one turn

$$Et = 2Econ = 41.4v$$

2- INDUCTION MACHINES

2-1 General Principle: -

The popularity of 3 phase induction motors on board ships is because of their simple, robust construction, and high reliability factor in the sea environment. A 3 phase induction motor can be used for different applications with various speed and load requirements. Electric motors can be found in almost every production process today. Getting the most out of your application is becoming more and more important in order to ensure cost-effective operations. The three-phase induction motors are the most widely used electric motors in industry. They run at essentially constant speed from no-load to full-load. However, the speed is frequency dependent and consequently these motors are not easily adapted to speed control. We usually prefer d.c. motors when large speed variations are required. Nevertheless, the 3-phase induction motors are simple, rugged, low-priced, easy to maintain and can be manufactured with characteristics to suit most industrial requirements. Like any electric motor, a 3-phase induction motor has a stator and a rotor. The stator carries a 3-phase winding (called stator winding) while the rotor carries a short-circuited winding (called rotor winding). Only the stator winding is fed from 3-phase supply. The rotor winding derives its voltage and power from the externally energized stator winding through electromagnetic induction and hence the name. The induction motor may be considered to be a transformer with a rotating secondary and it can, therefore, be described as a “transformer type” a.c. machine in which electrical energy is converted into mechanical energy. It has the following main advantages and also some disadvantages.

a) Advantages; -

1-it has very simple and extremely rugged, almost un breakable construction (especially squirrel cage type).

2-It´s cost is low and it is very reliable.

3-It has sufficiently high efficiency. in normal running conditions, no brushes are needed, hence frictional losses are reduced. it has a reasonably good power factor.

4-It´s starting arrangement is simple especially for squirrel cage type.

b) Disadvantages: -

1-It´s speed cannot be varied without sacrificing some of its efficiency

2-It´s speed decreases with increase in load.

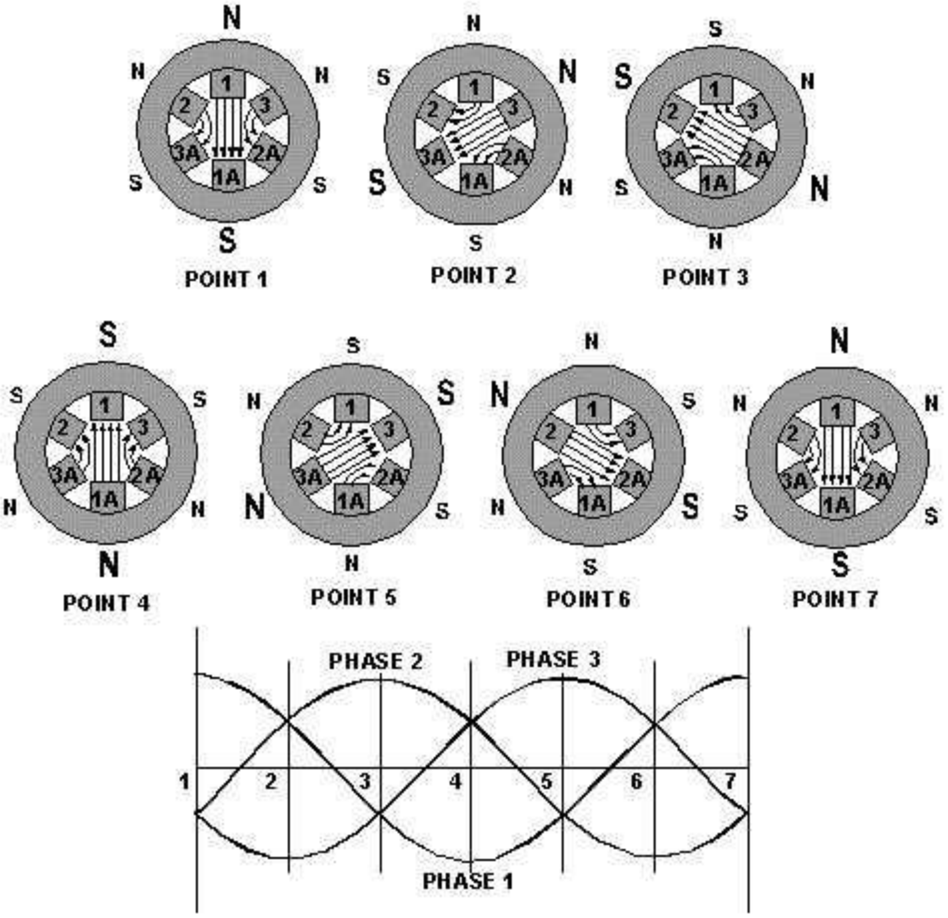
-from advantages and disadvantages we have they are simple ,reliable ,↓cost ,↑energy index ,rotor speed<syn. speed ,↓power factor.

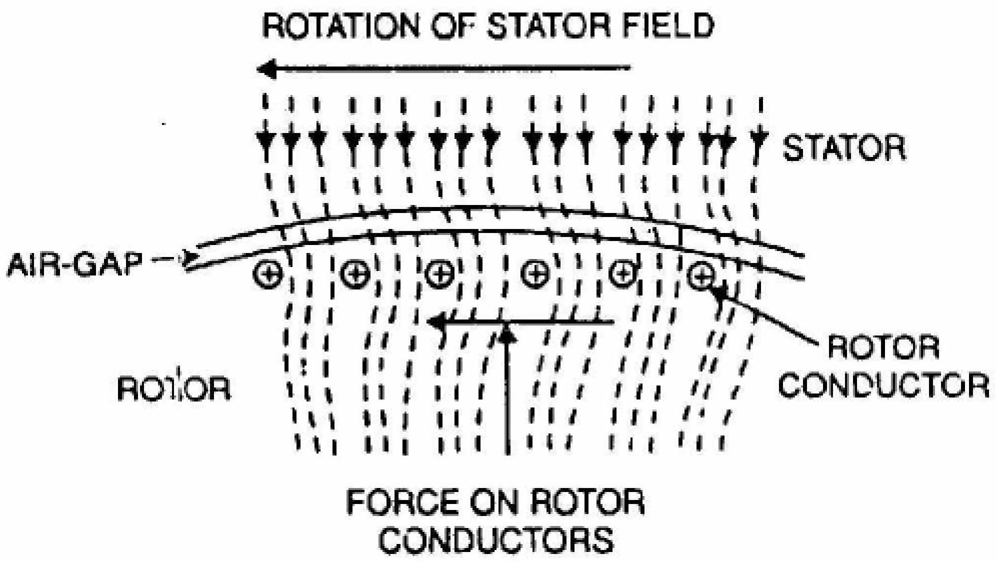
2-2 Operating Principles:

1) Energize the stator with three-phase voltage.

2) Currents in the stator winding produce a rotating magnetic field. This field revolves in the air gap.
3) The stator magnetic field links the rotor conductors through the air gap and voltage will be induced in the rotor conductors.
4) Currents in the rotor conductors will produce their own magnetic field which opposes the stator magnetic field.
5) The torque developed due to interaction of the stator and rotor magnetic fields pushes the rotor into rotation.
6) The direction of the rotation of the rotor is the same as the direction of the rotation of the revolving magnetic field in the air gap.

In a 3-phase induction motor, the three-phase currents Ia, Ib and Ic, each of equal magnitude, but differing in phase by 120°. Each phase current produces a magnetic flux and there is physical 120 °shift between each flux. The total flux in the machine is the sum of the three fluxes. The summation of the three ac fluxes results in a rotating flux, which turns with constant speed and has constant amplitude. Such a magnetic flux produced by balanced three phase currents flowing in three-phase windings is called a rotating magnetic flux or rotating magnetic field (RMF). RMF rotates with a constant speed (Synchronous Speed). Existence of a RFM is an essential condition for the operation of an induction motor. If stator is energized by an ac current, RMF is generated due to the applied current to the stator winding. This flux produces magnetic field and the field revolves in the air gap between stator and rotor. So, the magnetic field induces a voltage in the short circuited bars of the rotor. This voltage drives current through the bars. The interaction of the rotating flux and the rotor current generates a force that drives the motor and a torque is developed consequently. The torque is proportional with the flux density and the rotor 6 bar current (F=BLI). The motor speed is less than the synchronous speed. The direction of the rotation of the rotor is the same as the direction of the rotation of the revolving magnetic field in the air gap. However, for these currents to be induced, the speed of the physical rotor and the speed of the rotating magnetic field in the stator must be different, or else the magnetic field will not be moving relative to the rotor conductors and no currents will be induced. If by some chance this happens, the rotor typically slows slightly until a current is re-induced and then the rotor continues as before. This difference between the speed of the rotor and speed of the rotating magnetic field in the stator is called slip. It is unitless and is the ratio between the relative speed of the magnetic field as seen by the rotor the (slip speed) to the speed of the rotating stator field. Due to this an induction motor is sometimes referred to as an asynchronous machine.





2-3 Operating Modes: -

We have three modes for operation in I. Machines

Motoring: n= 0 - n   ,   s=1- 0

Generating: n: n_s -   (+∞)   ,s=0 - (-∞)

Braking: n: n - (+∞)   ,s=1-   (+∞)

mode	m	G	B
Driving	Self	P.M	P.M
Slip from to	0 +1	0 -∞	+1 +∞
speed	n<n1	n>n1	0<n<-∞
Rotating direction	→n →n_s	→n →n_s	→ n_s ←n

2-4 Rotating Rotor: -

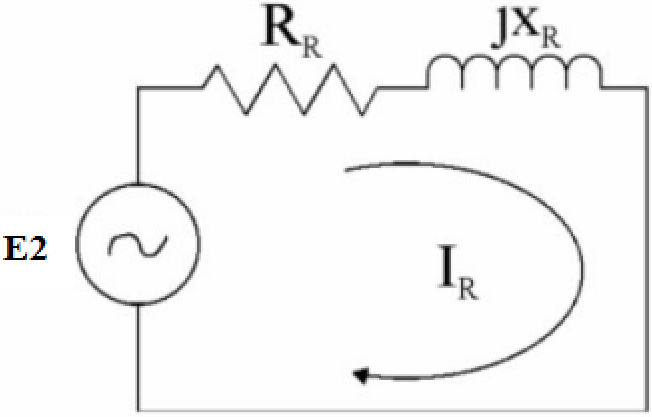
$$E2s = 4.44\, F2\, KW2\, T_2\, \emptyset M$$

$$= (4.44\, F1\, KW2\, T_2\, \emptyset M) * s = sE2$$

Where $S = f_2/f_1$

$$X2s = 2\pi f_2 L_2 = S(2\pi f_1 L_2) = SX_2$$

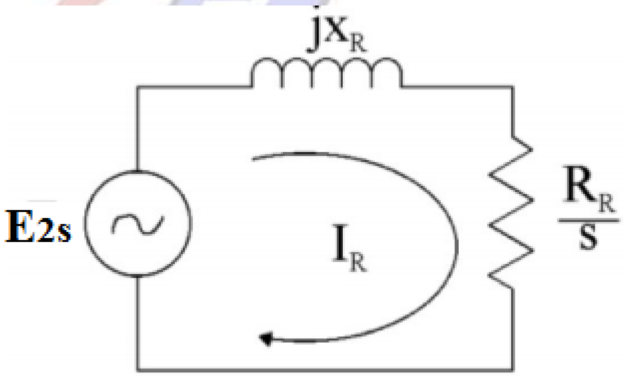
-R2S=R2 because it´s resistor



$$Z_2s = R_2s + jX_2s\,,\quad Z_2s = \sqrt{{R_2}^2 + {X_{2S}}^2}$$

$$I2S = \frac{E_2s}{Z_2s} = \frac{E_2s}{R_2 + jX_{2S}} = \frac{SE2}{R_2 + \mathrm{S}\mathrm{j}X_2}$$

$$= \frac{E2}{\frac{R_2}{\mathrm{S}} + \mathrm{j}X_2}$$



$$\varphi 2S = \tan^{-1}\frac{SX2}{R2} = \cos^{-1}\frac{R2}{Z2S}$$

$$I_{2S} = \left(\frac{E_2}{X_2}\right)\sin\varphi_{2S}\quad,\quad I_{2S} = I_2\quad,\varphi_{2S} = \varphi_2$$

2-5 Referred Values: -

F2=F2e

$$\frac{0.9\ M2\ I2\ KW2\ T2}{e}=\frac{0.9\ M1\ I1\ KW1\ T1}{e}$$

$$I2e=\left(\frac{m2kw2T2}{m1kw1T1}\right)I2=\frac{I2}{Ki}=I1$$

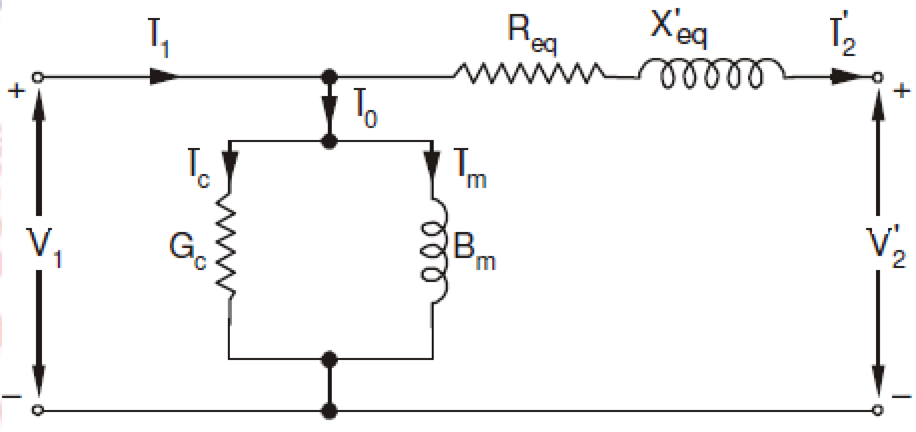
S2=S2e

$$m1E2I2=m1E2S\ I2S=m1E2S\left(\frac{M2KW2T2}{M1KW1T1}.I2\right)$$

$$E2e=\frac{KW1T1}{KW2T2}.E2=Ke\ E2=E1$$

$$I2^2R2=I2^2e\ R2e\quad ,m2\ I2^2R2=m1\ I2e\ R2e$$

$$R2e=\frac{m2}{m1}\frac{I2^2}{I2e^2}.R2=\frac{m2}{m1}(ki)(ki).R2$$



Where

$$\text{Ke}=\frac{m2}{m1}(ki)$$

$$\therefore R2e=Ke\ Ki\ R2$$

$$\frac{M2\ I2^2\ X2}{2}=\frac{M1\ I2e^2X2}{2}$$

$$X2e=\frac{m2}{m1}\left(\frac{I2}{I2e}\right)^2X2$$

$$=\left(\frac{m2}{m1}ki\right)ki\ X2$$

$$=ke\ ki\ X2$$

In general, ke=ki=k then

$$I2e=\frac{I2}{K}\qquad ,E2e=E2k\qquad ,R2e=k^2R2\ \ ,X2e=k^2x2$$

For squirrel-cage motor.

$$m2=\frac{s2}{p2}\qquad ,T2=0.5\ \ ,Kw2=1$$

2-6 Phasor Diagram: -

for ideal motor

$E2 \rightarrow E1 \rightarrow \emptyset m$ Z1=0 pcu≠0

E1 = E2 $at\ S = 1$

F1=-F2 , I1=-I2 ,IM=0

-FOR ACTUAL MOTOR

Excitation mmF Fe=F1+F2

Ie=I1+I2e , *Ie=Io* ,Ie=Im+Ii -*Fig.1*

V1=-E1+I1(R1+Jx1) -*Fig.2*

$$v2 = -E2S + I2e\ Z2S$$

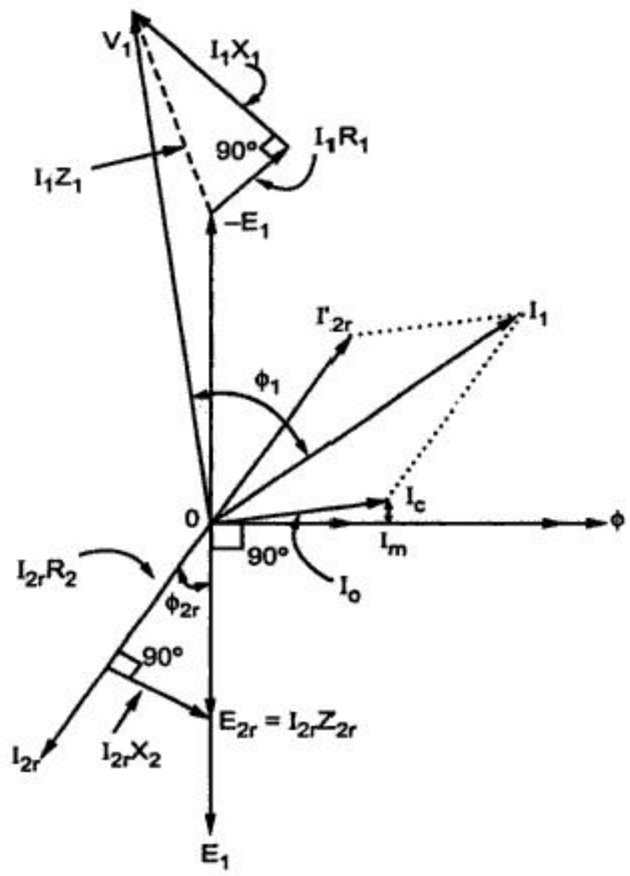
$$= -SE1 + I2e(R2e + JSX2e)$$

$$= -E1 + I2e\left(\frac{R2e}{s} + jx2e\right)$$

$$\frac{R2e}{s} = \frac{R2e}{s} + R2e - R2e$$

$$= R2e + \left(R2e\ \frac{1-S}{S}\right)$$

$$I2e^2\frac{R2e}{s} = I2e^2\ R2e + I2e^2\ R2e\ \frac{1-s}{s} = pcu2e + pm$$

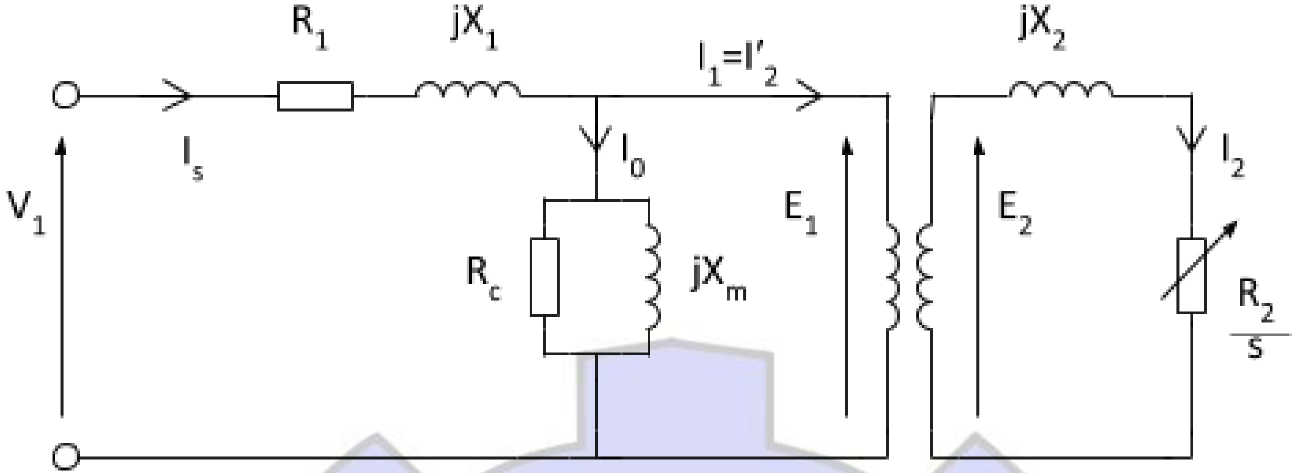


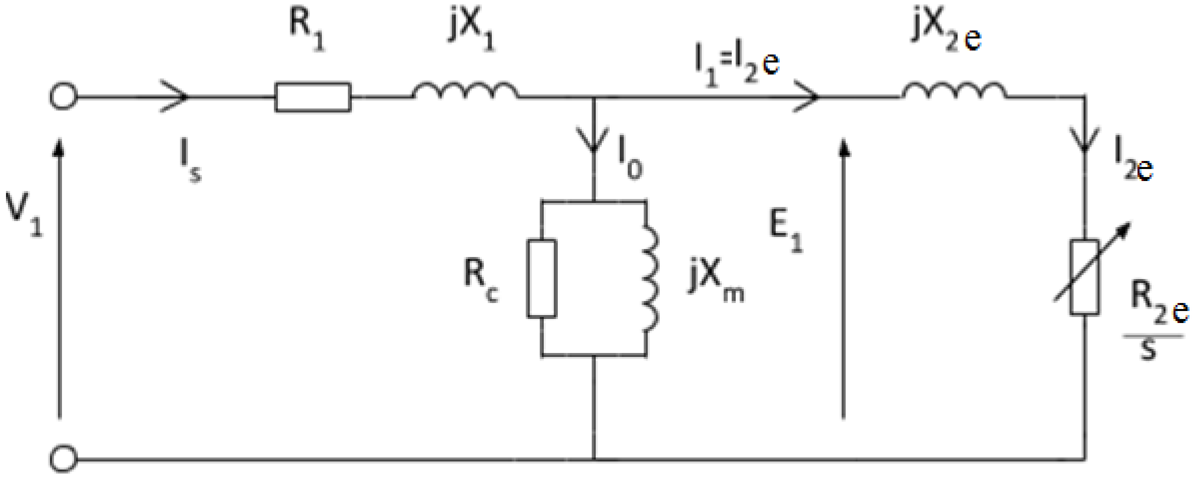
2-7 Equivalent Circuit: -

$E1 = IeZe$; $E1 \propto Ie$; $Ze = Exitation\ impedance$

$$\text{Ye} = \frac{1}{ze} = \frac{1}{Ri} + \frac{1}{jXm} = Gi - jBm = Admittance$$

$Gi = conductance \ \ , Bm = susceptance$





$$Ie = \frac{Ei}{Ze} = E1\,\gamma e = E1\,Gi - jE1Bm$$

$Ie = Ii + Im$

We have: -

$v1 = -E1 + I1Z1$

$v1 = Ie\ Ze + I1Z1$

$0 = -E1 + I2e\ Z2es = Ie\ Ze + I2e\ Z2es$

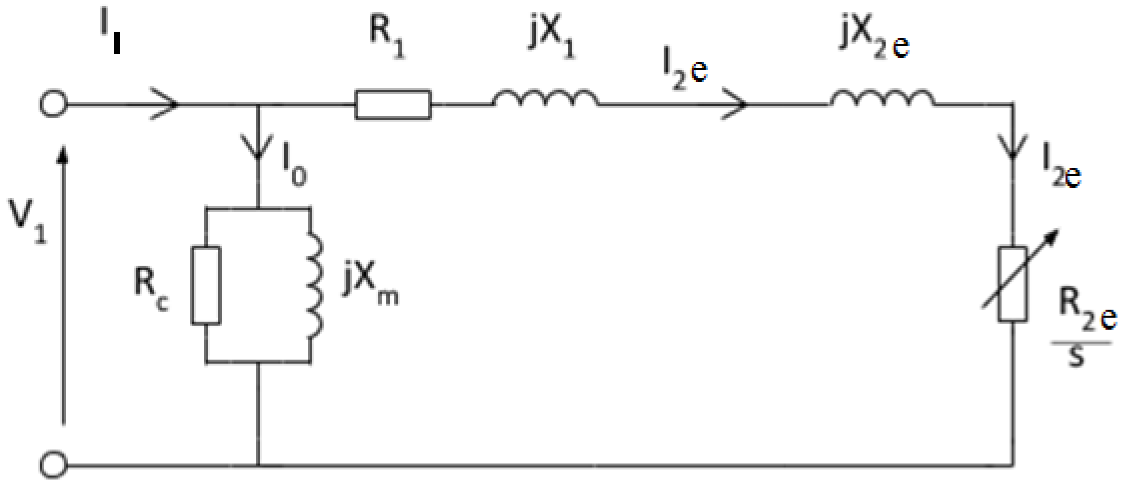
$I2e = Ie - Ii$

By use this equation we will have

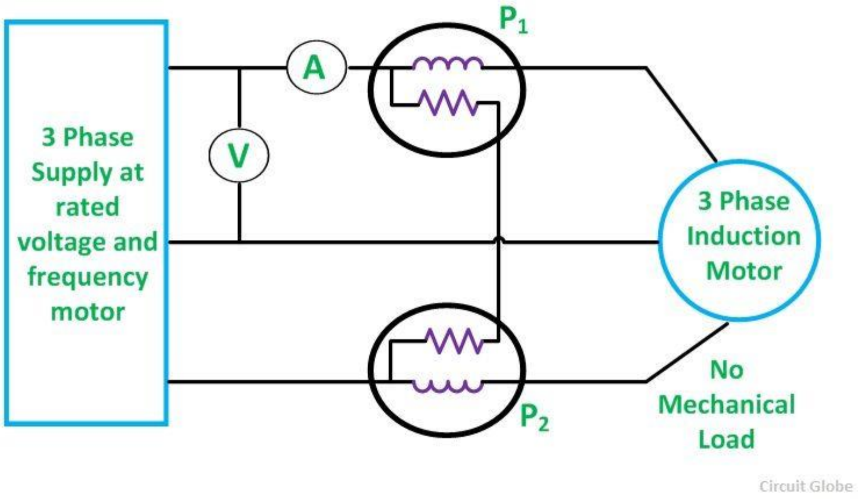
$$Ie = I1\frac{z2e}{ze+z2es}$$

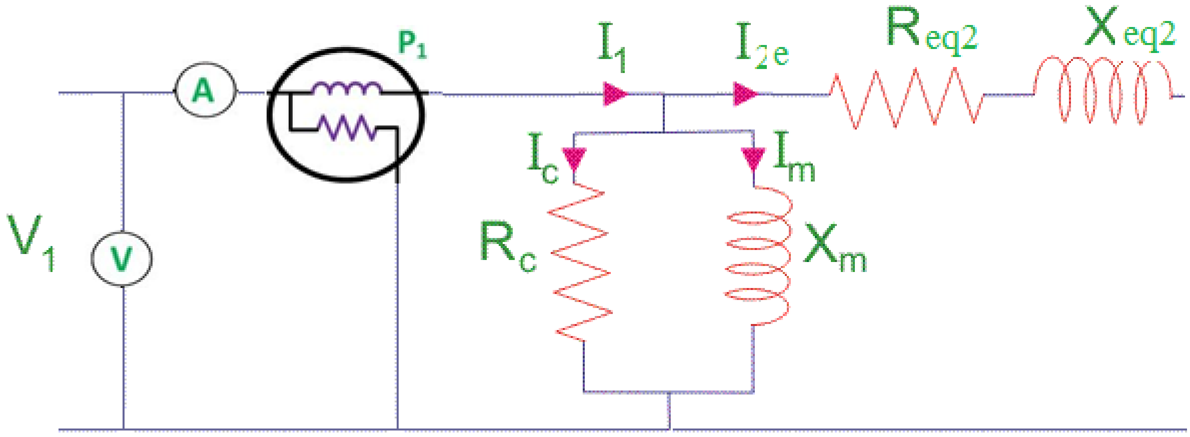
$$\boxed{V1 = I1\left(Z1+\frac{Z2es.Ze}{Ze+Z2es}\right) = I1Zt}$$

Approximate Equivalent cct



***No –load test**





Vo, Po, Io

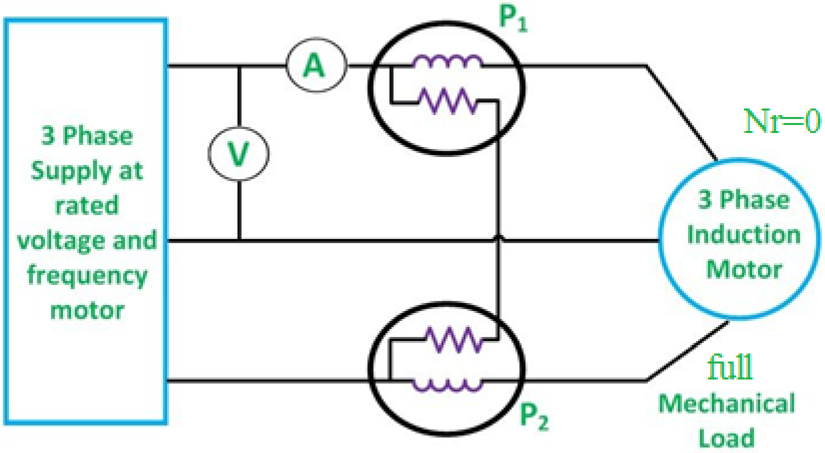
$$I1 = Io \quad ,V1 = Vo \quad ,Ie = I1 = Io$$

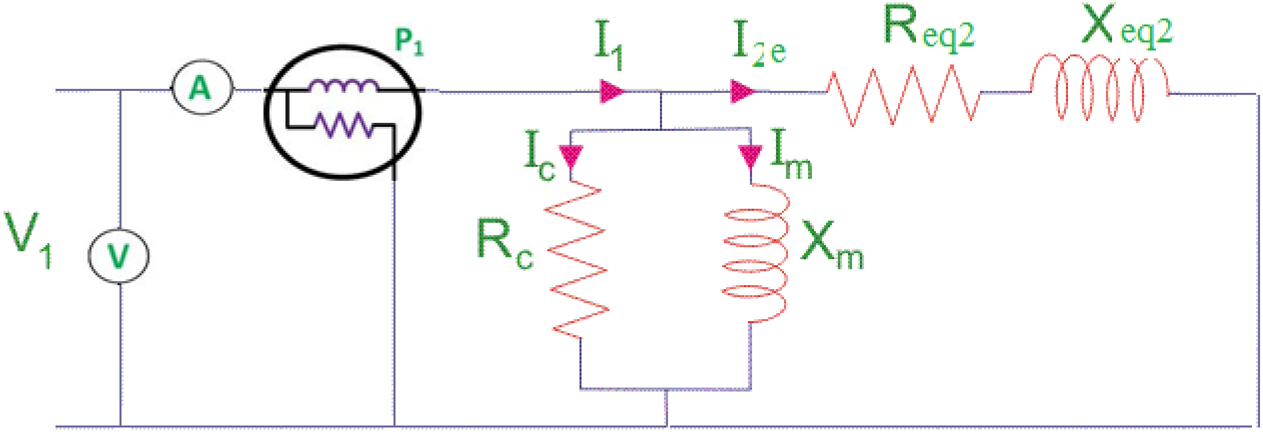
$$Po = vo\, Ii = vo(vo\, Gi) = vo^2 Gi$$

$$Gi = \frac{po}{vo^2} \qquad ; \quad Ii = vo\, Gi$$

$$Im = \sqrt{Io^2 - Ii^2} \quad ; \quad Bm = \frac{Im}{vo}$$

*short –circuit test





vsc, Isc, psc

$$zsc = z1 + z2e \quad = Rsc + jXsc = \frac{vsc}{Isc}$$

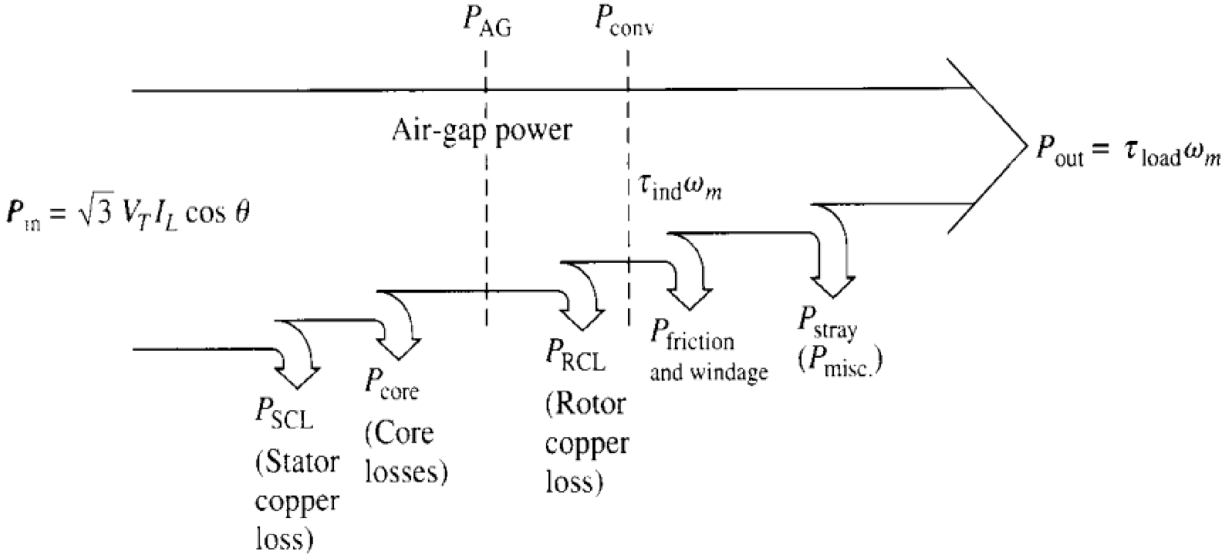
$$Rsc = \frac{Psc}{{I_{sc}}^2} = R1 + R2e$$

$$Xsc = X1 + X2e = \sqrt{Zsc^2 - Rsc^2}$$

$$R2e = Rsc - R1$$

$$X2e \cong X1 = \frac{Xsc}{2}$$

2-8 Power and Energy Diagram: -



$$P1 = m1\, I1\, V1\ \cos\theta 1 = 3I1\, V1\ \cos\theta 1 = \sqrt{3}\, I_L V_L \cos\theta\, 1$$

$$P_M = I2e^2 R\mu = I2e^2\ R2e\ \frac{1-S}{S}\ \ , I2e = \frac{E2e}{Z2es}$$

$$P_M = E2e(1-s)I2e\ \frac{\frac{R2e}{s}}{Z2e}\ = (1-s)I2e\ E2e\cos\theta 2e$$

Since $.\frac{R2e/s}{Z2e}$

$$P_M = \ I2e\ E2e\cos\theta 2 - S\ E2e\ I2e\ \cos\theta 2e = Pem - Pcu2$$

$$P_M = Pem - sPem = (1-s)Pem$$

$$Pcu2 = I2e^2 R2e = \frac{E2e}{Z2e} I2e\ R2e = E2e\ I2e\ \frac{R2e}{Z2es}$$

$$= sE2e\ I2e\ \frac{\frac{R2e}{s}}{Z2es} = \mathrm{s\ E2e\ I2e\ \cos\theta 2}$$

$$Pem = \ P_M + \ Pcu2 = I2e^2 \frac{R2e}{s} = I2e^2\left(R2e + R2e\ \frac{1-s}{s}\right) = \frac{Pcu2}{s}$$

$$P2 = P_M - (Pfw + Padd); Pem = P1 - Pcu1 - Pfe$$

$$P_M = Pem - \ Pcu2 = P1 - Pcu1 - Pfe - Pcu2$$

$$P2 = P1 - (Pcu1 + Pcu2 + Pfe + Pfw + \ Padd)$$

$$Tem = \frac{Pem}{\omega 1} = \frac{60}{2\pi ns}Pem = \frac{P}{2\pi F1}Pem$$

$$Tem = \frac{P_M}{\omega 2} = \frac{60}{2\pi n}P_M = \frac{P}{2\pi F2}P_M$$

$$Pcu2 = Pem - P_M = Tem(\omega 1 - \omega 2) = s(\omega 1\, Tem) = sPem$$

Where: $s = \frac{\omega 1 - \omega 2}{\omega 1}$

$$Tem = \frac{Pcu2}{s\omega 1}, \quad s = \frac{Pcu2}{\omega 1\, Tem} = \frac{Pcu2}{Pem} = \frac{I2e^2\, R2e}{I2e^2\, R2e/s}$$

$$P_M = I2e^2\, R2e\, \frac{1-s}{s} = Pcu2\, \frac{1-s}{s} = sPem\left(\frac{1-s}{s}\right) = (1-s)Pem$$

Ex1) A3-phase ,4pole,50HZ IM ,the impedance of the star connected rotor winding is(0.2+j3)per phase the no-load induced emf between slip-rings is 220v ,find the rotor current and power factor at rated.

Sol:

$$n1 = ns = \frac{60F}{P} = \frac{60 \times 50}{2} = \frac{3000}{2} = 1500\, rpm$$

$$s = \frac{n1 - n}{n1} = \frac{1500 - 1455}{1500} = 0.03$$

$$E2 = \frac{220}{\sqrt{3}} = 127V$$

$$E2S = SE2 = 0.03 \times 127 = 3.81V$$

$$Z2S = \sqrt{R2^2 + (SX2)^2} = \sqrt{0.2^2 + (0.03 \times 3)^2} = 0.22\Omega$$

$$I2 = \frac{E2S}{Z2S} = \frac{3.81}{0.22} = 17.32A$$

$$\cos\theta 2 = \frac{R2}{Z2} = \frac{0.2}{0.22} = 0.909$$

Ex2) A 3-phase ,4-pole,380v,50HZ IM with rotor impedance of (0.1+j0.6) Ω/phase, both winding is λ-connected with T1/T2=1.5. find the stator current if the excitation current is(1-j10) A, and motor speed is 1440 rpm, neglect the primary impedance(Z1=0)

SOL:

$$n1 = \frac{60F1}{P} = 1500rpm, s = \frac{1500 - 1440}{1500} = 0.04$$

$$R2e = 0.1 \times 1.5^2 = 0.225\Omega\ , X2e = 0.6 \times 1.5^2 = 1.35\Omega$$

$$Vphas = \frac{Vline}{\sqrt{3}} = \frac{380}{\sqrt{3}} = 220v\ , Z2e = \frac{R2e}{s} + jX2e = \frac{0.225}{0.04} + j1.35$$

$$I2e = \frac{v}{z2e} = \frac{220}{5.625 + j1.35} = 37 - j8.87A$$

$$I1 = Ie + I2e = (1 - j10) + (37 - j8.87) = 38 - j18.87$$

$$I1 = 42.43$$

Ex3) A3-phase ,6-pole,440v ,50HZ IM. the parameters of its star connected winding are,R1=R2e=0.1 ,X1=X2e=0.4 ,Ye=0.008-j0.06Ω/phase. find the current and power factor at 50% the rated load at which the slip is 0.04.use the exact equivalent cct.

Sol:

$$Rm = R2e\frac{1 - s}{s} = 0.1\ \frac{1 - 0.04}{0.04} = 2.4\Omega$$

$$Vp = \frac{440}{\sqrt{3}} = 254v$$

$$y2e = \frac{1}{z2e} = \frac{1}{2.5 + j0.4} = 0.39 - j0.06$$

$$yab = ye + y2e = (0.008 - j0.06) + (0.39 + j0.06) = 0.398 - j0.66$$

$$Zab = \frac{1}{yab} = \frac{1}{0.398 - j0.66} = \frac{0.398 - j0.66}{0.398^2 - 0.66^2} = 0.67 + j1.11$$

$$Zt = Z1 + Zab = (0.1 + j0.4) + (0.67 + j1.11) = 0.77 + j1.51$$

$$I1 = \frac{Vphase}{Zt} = \frac{254}{0.77 + j1.51} = 68 - j133.5$$

$$I1 = 150A \ , \cos\theta 1 = \frac{Ia}{I1} = \frac{68}{150} = 0.45$$

Ex4) A3-phase, 4pole, 50H$_Z$ IM, at full-load delivers,T$_2$=120N.M torque at 0.03slip.find the efficiency if the total stator losses are (0.1P$_2$)and P$_{fw}$=0.01P2

Sol:

$$n1 = ns = \frac{60F1}{P} = \frac{60 \times 50}{2} = 1500\, rpm$$

$$s = \frac{n1 - n}{n1} = \frac{1500 - n}{1500} = 0.03 \rightarrow \mathrm{n} = 1455 \text{ rpm}$$

$$or\ n = (1 - s)n1 = (1 - 0.03) \times 1500 = 1455rpm$$

The useful out put power is

$$P2 = \omega.T = \frac{2\pi n}{60} \times T2 = 2\pi \frac{1455}{60} \times 120 = 18275\omega$$

$$P\mu = P2 + Pfw = 1.01P2 = 1.01 \times 18275 = 18458\omega$$

$$Pem = P\mu \frac{n1}{n} = 18458 \times \frac{1500}{1455} = 19029$$

$$or\ Pem = \frac{P\mu}{1 - \mathrm{s}} = \frac{18458}{1 - 0.03} = 19029\omega$$

The total input power to the motor is :

$$P1 = Pem + Pstator = 19029 + 0.1 \times 18275 = 20932\omega$$

And efficiency

ɳ $= \frac{18275}{20932} = \frac{P2}{P1}$

3-Motor Circle Diagram

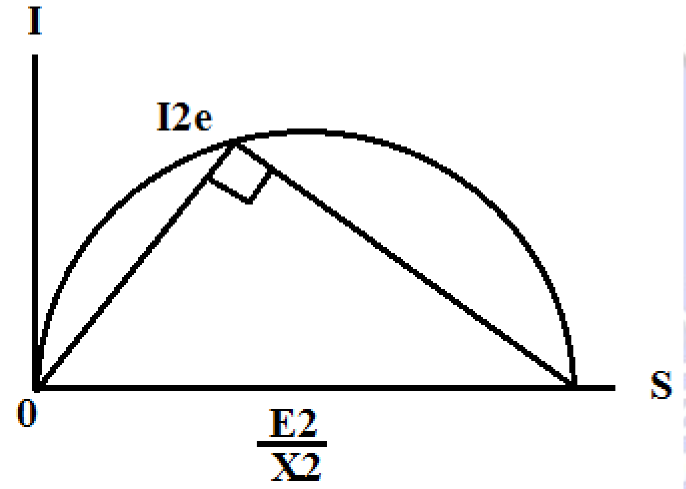
3-1 Rotor Circle Diagram: -

$$I2e=\frac{E2e}{\sqrt{\left(\frac{\mathrm{R2e}}{\mathrm{s}}\right)^{2}+X2e^{2}}}\quad \dots\dots\dots\dots\dots. eq(1)$$

$$I2e, E2e\ ,\cos\varphi 2=function\ to\ S\ \ F(s)$$

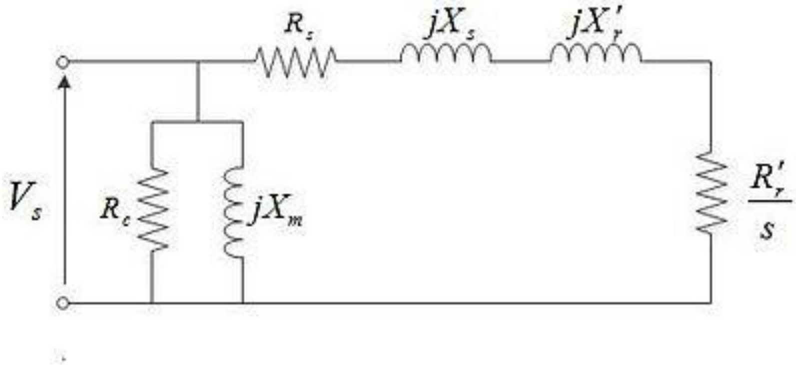
$$OD=\frac{OA}{\sin\varphi 2}=\frac{I2e}{\frac{\mathrm{X2e}}{\mathrm{Z2s}}}=\frac{E2S}{\mathrm{Z2S}}\times\frac{Z2S}{\mathrm{X2S}}=\frac{E2S}{\mathrm{X2S}}=\frac{SE2}{\mathrm{SX2}}=\frac{E2}{\mathrm{X2}}=CONSTANT$$

S=0 ,I2s=0



Circle Diagram for the approximate equivalent cct.

It is clear that the circuit to the right of points ab is similar to a series circuit ,having a constant voltage and reactance (X1+X2e)but variable resistance (corresponding to different values of slip s)

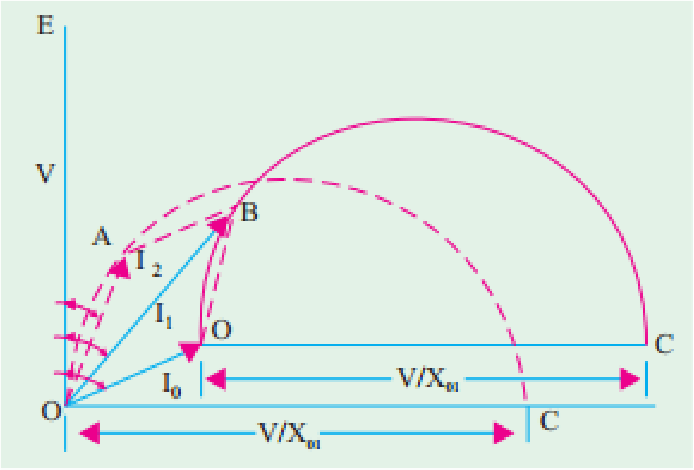


Hence, the end of current vector for I2e will line On a circle with a diameter of $V/(x_1+x_{2e})$.In fig.(2) I_{2e} is the rotor current referred to stator, $I_e=I_o$= no load current or exciting current and I1 is the total stator current and is the

When I_{2e} is lagging and $\varphi_2=90^\circ$, then the position of vector for I_{2e} will be along OC, right angles to the voltage vector OE, for any other value of φ_2, point A will be move along the circle shown dotted.

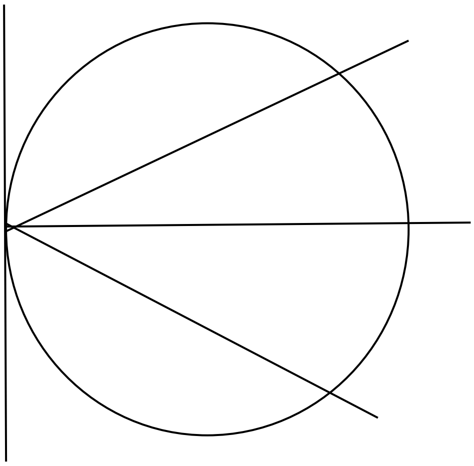
The exciting current I_e is drawn lagging V by an angle φ_e if conductance Gi and saucepans BM of the exciting circuit are assumed constant, then I_e and φ_e are also constant.

The end of current vector for I_1 is also seen to lie on another circle which is displaced from the dotted circle by an amount Ie. its diameter is still V/X_1+X_{2e} and is parallel to the horizontal axis OC. hence, we find that if an induction motor is tested at various loads, the locus of the end of the vector for the current (drawn by it) is a circle.



Fig(2)

Complete Circle :-



Motoring s=0 →+1

Braking s=1 →+∞

Generating s=0 → -∞

3-2 Motor Mechanical Characteristics

-Torque from electromagnetic forces:-

$$Pem = m1\ E2e\ I2e\ \cos\varphi 2 \ldots\ldots\ldots(1)$$

$$E2e = enduce\ emf\ in\ rotor\ coils$$

$$E2e = 4.44\ F1\ T1\ K\omega 1\emptyset \ldots\ldots\ldots(2)$$

$$\cos\varphi = Power\ Factor$$

$$\cos\varphi = \frac{\mathrm{R2e}}{\mathrm{Z2e}} = \frac{\mathrm{R2e}}{\sqrt{\mathrm{R2e}^2 + (\mathrm{SX2e})^2}} \ldots\ldots\ldots(3)$$

We will have torque equ**.**

$$Tem = \frac{Pem}{\omega 1} = \frac{p}{2\pi f1}.Pem \ldots\ldots\ldots(4)$$

$$= \frac{m1\ p}{2\pi f1}(4.44\ f1\ T1\ K\omega 1)I2e\ \Phi\ \cos\varphi 2$$

$$= \frac{1}{\sqrt{2}}(m1\ K\omega 1\ T1\ I2e\cos\varphi 2)P\emptyset$$

$$Tem = KT\ \emptyset m\ I2e\ \cos\varphi 2 \ldots\ldots\ldots(*)$$

$$* kT = \frac{1}{\sqrt{2}} p\, m1\, k\omega1\, T1$$

المعادلة(*)تعبر عن عزم الدوران في جميع المكائن الكهربائية وهي تعني بأن مقدار العزم يتناسب طرديا مع الفيض المغناطيسي($\emptyset$)ومع المكونة الفعالة لتيار الدوار.

وان مكونه التيار تعتمد على مقدار وزاوية الطور لهذا التيار، وتعتمد زاوية الطور على طبيعة ممانعة لفيفه الدوار

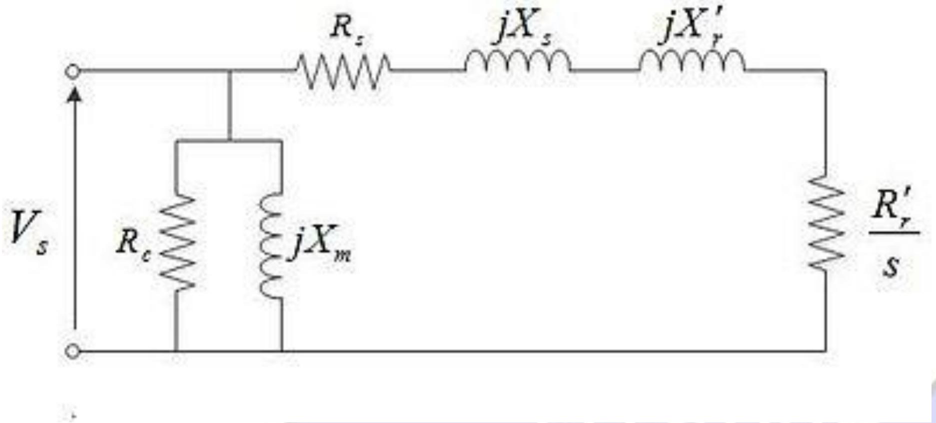
$$Tem \propto \emptyset, I2e, \cos\varphi$$

إن المشكلة في هذه المعادلة هي أن العزم فيها هو داله لثلاث مقادير هي($\emptyset$)و($I2e$)و($cos\varphi 2$) وكلها مقادير متغيره لها علاقة بمقدار الانزلاق وحمل المحرك لهذا يفضل عمليا الاستعانة بمعادله الدائرة المكافئة للتعبير عن عزم الدوران.

$$Tem = \frac{Pem}{\omega1} = \frac{m1}{\omega1} I2e^2 \frac{R2e}{S}$$

3-3 Torque From Electromagnetic Power

-For Approximate Equivalent cct.



$$Z2t = \left(R1 + \frac{R2e}{s}\right) + j(X1 + X2e),\ \ V1 = E1$$

$$I2e = \frac{V1}{Z2t} = \frac{V1}{\sqrt{\left(R1 + \frac{R2e}{s}\right)^2 + (X1 + X2e)^2}}.$$

$$Pem = \frac{m\, I2e^2\, R2e}{s}$$

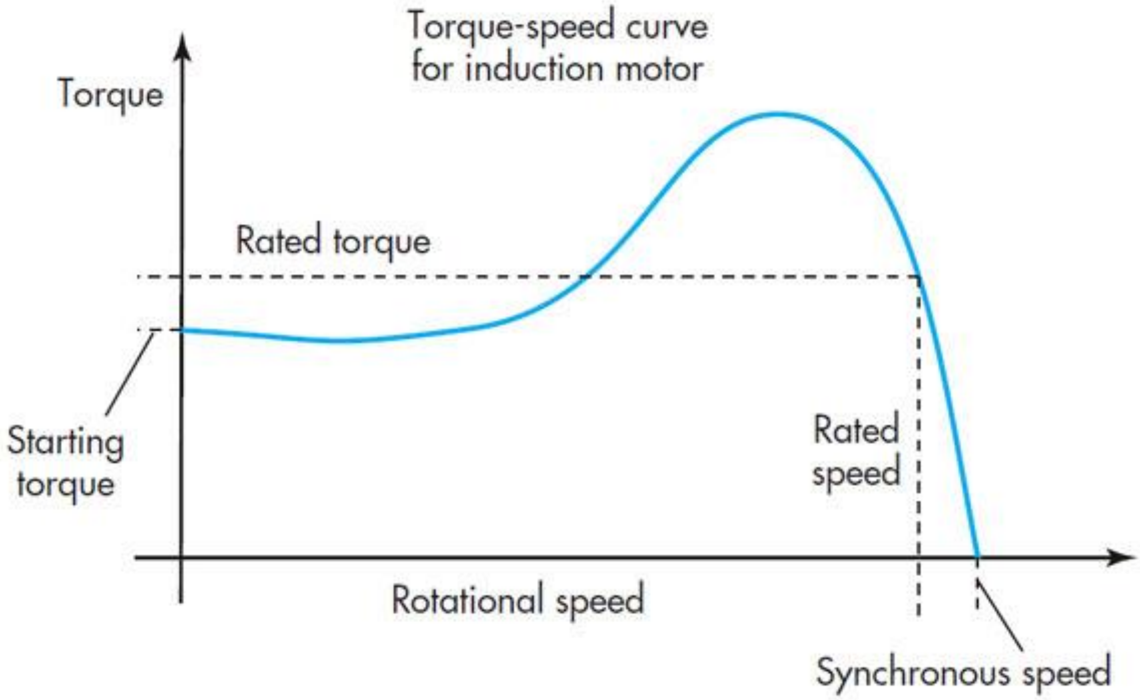
$$Tem = \frac{m1\, Pem}{\omega 1} = \frac{m1}{\omega 1}\left[\frac{v1^2 . \frac{R2e}{s}}{\left(R1 + \frac{R2e}{s}\right)^2 + (X1 + X2e)^2}\right]$$

T_{em} will variable with (V1,R2e,S)

At S=1 and V1=const.→Tα R2e

عند ثبوت مقدار الجهد المسلط فأن العزم يتناسب طرديا مع مقدار مقاومه لفيض الدوار عند بدء الحركة(s=1).ولزيادة العزم ينبغي زيادة مقدار المقاومة.

في الحالات الاعتيادية يكون مقدار الجهد المسلط ومقدار المقاومة في الدوار ثابتين فأن العزم يتغير بتغير الانزلاق(s)ويعطي منحني(Torque slip curve)



$$Tem = \frac{Pem}{\omega 1} = \frac{m1}{\omega 1}\left(\frac{V1^2\,\frac{R2e}{s}}{\left(R1+\frac{R2e}{s}\right)^2 +}\right)$$

Multiply by s^2/s^2 for 3phase motor

$$Tem = \frac{3}{\omega 1}\left(\frac{V1^2 \;\; R2e\,S}{(SR1+R2e)^2 \;\; + S^2(X1+X2e)^2}\right)$$

If ,s=0 then Tem=0

For s↓↓ ($s<s_m$) then SR1 and $S^2(X_1+X_{2e})^2$ are neglected then we will have

$$Tem = \left(\frac{3\,V1^2}{\omega 1\;R2e}\right)s$$

The torque is directly proportional to the slip and the torque/slip curve is a straight line.

For S↑↑($S>S_m$) neglect $SR_1=0$ then

$$Tem = \left(\frac{3\,V1^2 R2e}{\omega 1\;(\mathrm{X1}+\mathrm{X2e})^2}\right).\frac{1}{s}$$

Then the torque is inversely proportional to the slip and the torque is continuously droping starting from critical slip at S=∞ then the $Tem=0$

3-4 Maximum Torque: -

Taking the derivative of Tem by S (dTem/ds) and equating this to zero (dt/ds =0)then the slip at max. torque(critical slip) can be obtained:-

$$\boxed{sm=\pm\frac{R2e}{\sqrt{R1^2+(X1+X2)^2}}}$$

If neglecting the stator risis.(R_1=0)then

$$\boxed{sm=\pm\frac{R2e}{\sqrt{(X1+X2)^2}}=\pm\frac{R2e}{\mathrm{xsc}}}$$

Substituting S_m in the torque equation in (Tem)then

$$\boxed{Tm=\pm\frac{3V1^2}{\omega 1}\times\frac{1}{2(\mathrm{R1}\pm\sqrt{\mathrm{R1}^2+(\mathrm{X1}+\mathrm{X2})^2}}}$$

Neglecting the R_1=0 then the maximum torque is

$$Tm=1.5\ \frac{V^2}{\omega 1\ XSC}=0.2388P\ \frac{V^2}{f1\ XSC}$$

The max. torque value is inversely proportional to the short circuit reactance(Tα 1/XSC)and it is independent of (R_{2e}):that mane it's no effect of R_{2e} on T_m value.

To have max. torque at starting: $Ts=T_m$

Then $S_m=1=R_{2e}/X_{SC}$ Or $R_{2e}=X_{sc}=X_1+X_{sc}$

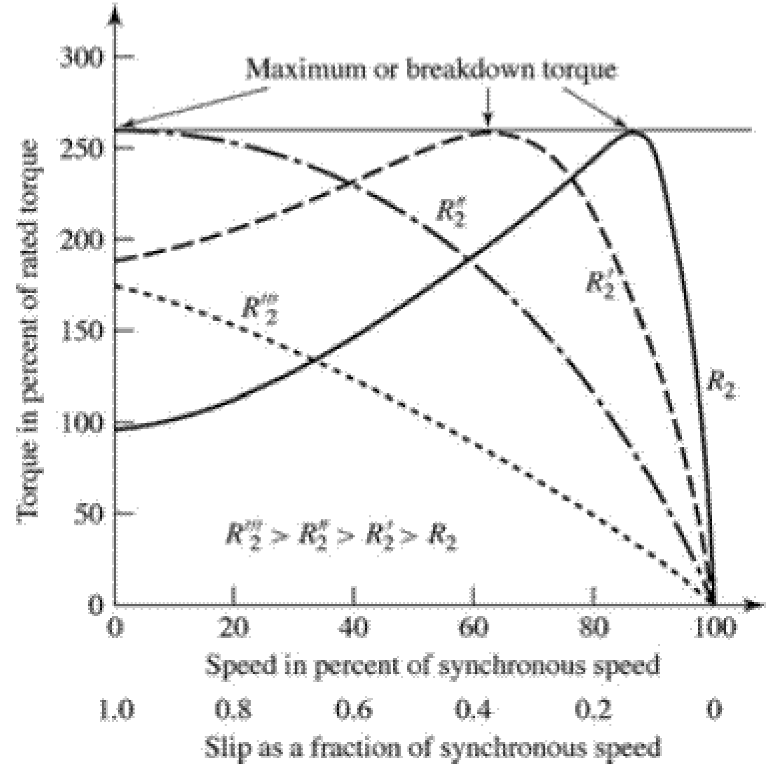
This means that $\varphi_2=45^\circ$ and cosφ2=0.707

3-5 Starting Torque:-

At starting S=1, substituting this slip value is torque equation then.

$$Ts = \frac{3V1^2}{\omega 1} \cdot \frac{R2e}{(R1 + R2e)^2 + (x1 + x2e)^2}$$

Usually if is required that Ts=Tm then Sm=1 or R2e=X2e if should be rotated that R2e/s= constant always or $\frac{R2}{S} = \frac{R2'}{S'} = \frac{R2'''}{S'''} = \frac{R2''''}{S''''}$



3- 6 Effect of Applied Voltage:-

From approximate equivalent cct and assuming Z1=0 then

$$Tm=\frac{3}{\omega 1}.\frac{V1^2R2e}{(R2e)^2+(SX2e)^2}\qquad since\ \ T1=T2=T$$

Then

$$\frac{V1^2\ \ S1\ R2e}{(R2e)^2+(S1X2e)^2}=\frac{V2^2\ S2\ R2e}{(R2e)^2+(S2X2e)^2}$$

$$OR\ \left(\frac{V1}{V2}\right)^2=\frac{S2}{S1}(\frac{R2e^2+(S1\ X2e)^2}{R2e^2+(S2\ X2e)^2}$$

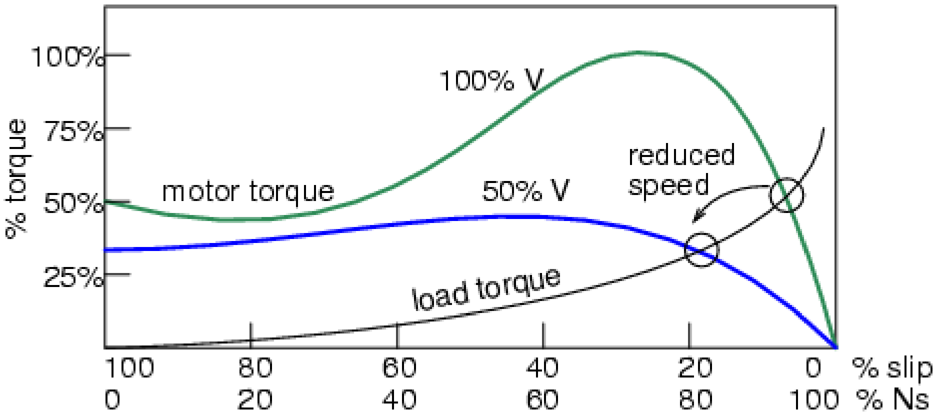
Since slip variation will not effect the value in the rotor coil of machine then.

$$\frac{T1}{T2}=\left(\frac{V1}{V2}\right)^2=\frac{S2}{S1}\qquad Since\ S1\ X2e=S2\ X2e$$

If $V_2=0.75V_1$ then T2=0.44 T1

That mane the voltage variation ΔV must be in rang of (10-(-5))%

ΔV=+10% , -5%



3-7 Ratio of Max To Rated Torque:-

$$Tr = \frac{3V^2}{\omega 1}.\frac{R2e/sr}{(R1 + R2e/sr)^2 + (x1 + x2)^2}$$

$$Tm = \frac{3V^2}{\omega 1}.\frac{\frac{R2e}{sm}}{\left(R1 + \frac{R2e}{sm}\right)^2 + (x1 + x2)^2}$$

Kmr or $\mu m = \frac{Tm}{Tr}$

$$\mu m = \frac{\left(R1 + \frac{R2e}{sr}\right)^2 + (X1 + X2e)^2}{\left(R1 + \frac{R2e}{sm}\right)^2 + (X1 + X2e)^2}.\frac{sr}{sm}$$

$$Sm = \frac{R2}{\sqrt{R1^2 + (X1 + X2)^2}} \rightarrow R1^2 + (X1 + X2)^2 = \left(\frac{R2}{Sm}\right)^2$$

$$(X1 + X2)^2 = \left(\frac{R2}{Sm}\right)^2 - R1^2$$

$$\mu m = \frac{R1^2 + 2R1\frac{R2}{Sr} + \left(\frac{R2}{Sr}\right)^2 + \left(\frac{R2}{Sm}\right)^2 - R1^2}{R1^2 + 2R1\frac{R2}{Sr} + \left(\frac{R2}{Sr}\right)^2 + \left(\frac{R2}{Sm}\right)^2 - R1^2}.\frac{sr}{sm}$$

$$= \frac{2R1R2 + \frac{R2^2}{Sr} + sr\left(\frac{R2}{Sm}\right)^2}{2R1R2 + \frac{R2^2}{Sm} + \frac{R2^2}{Sm}} \times \frac{R2^2}{R2^2} divided$$

$$= \frac{\frac{2R1}{R2} + \frac{1}{Sr} + \frac{Sr}{Sm^2}}{\frac{2R1}{R2} + \frac{1}{Sm} + \frac{1}{Sm}} \times \frac{sm}{sm}$$

$$\mu m = \frac{\frac{sm}{sr} + \frac{sr}{sm} + 2sm\frac{R1}{R2}}{2 + 2Sm\frac{R1}{R2}}$$

$$\mu m = \frac{\frac{sm}{sr} + \frac{sr}{sm} + Q}{2 + Q} \qquad Q = 2Sm\frac{R1}{R2}$$

$$Z1 = 0, \quad Q = 0$$

$$\frac{Tm}{Tr} = \frac{\frac{sm}{sr} + \frac{sr}{sm}}{2} = 2\frac{Tm}{Tr} = \frac{Sm^2 + Sr^2}{Sm\,Sr}$$

$$\frac{Tm}{Tr} = \frac{Sm^2 + Sr^2}{2Sm\,Sr}$$

μm= 1.7 …….3

3-8 Ratio of Starting To Rated Torque:-

$$T = \frac{3}{\omega 1}\ I2e^2\frac{R2e}{s}\ in\ general$$

$$at\ starting\ \ Ts = \frac{3}{\omega 1}I2s^2R2e = \frac{Pcu2}{\omega 1}\ldots\ldots\ldots(1)$$

$$at\ rated\ condition\ \ Tr = \frac{3}{\omega 1}\ I2r^2\frac{R2e}{sr}$$

$$then\ \mu s\ \ or\ \ Ksr = \frac{Ts}{Tr} = Sr\ \left(\frac{I2s}{I2r}\right)^2$$

3-9 Ratio of Starting To Maxi Torque:-

$$Tm = \frac{3}{\omega 1}\cdot\frac{V1^2 sm\ R2e}{R2e^2 + (sm\ X2e)^2} \qquad assuming\ that\ Z1 = \qquad also\ R2e = Sm\ \ X2e$$

$$then\ \ Tm = \frac{3V1^2}{\omega 1}\cdot\frac{sm(sm\ X2e)}{2(sm\ X2e)^2} = \frac{3V1^2}{2\omega 1X2e}$$

$$similarly\ \ \ Ts = \frac{3}{\omega 1}\cdot\frac{V1^2\ R2e}{R2e^2 + (\ X2e)^2} \qquad , since\ \ \ s = 1$$

$$then \quad \mu sm \quad or\ Ksm = \frac{Ts}{Tm} = \frac{2R2e\,X2e}{R2e^2 + X2e^2} = \frac{2R2e\,\frac{X2e}{X2e^2}}{\frac{R2e^2 + X2e^2}{X2e^2}}$$

$$\mu sm = \frac{2sm}{1 + sm^2} \qquad (0.3 - 0.7)$$

Ex 1) A 3-phase ,5.5kw,4-pole,50HZ,1455 rpm IM. its stand still rotor impedance is Z2=0.8+j4 and rotor short circuit current is 60A.neglect the friction and winding losses and find 1)Ts/Tr ,2)Ts/Tm

Sol:-

$$n1 = \frac{60F}{P} = 1500rpm \quad , \quad Sr = \frac{no - n}{no}, sr = \frac{1500 - 1455}{1500} = 0.03$$

$$sm = \frac{R2}{X2} = \frac{0.8}{4} = 0.2 \quad Pcu2 = Pm\ \frac{sr}{1 - sr} = 5500 \,.\frac{0.03}{1 - 0.03} = 170w$$

$$Pcu2 = 3I2r\ \ R2 \quad or\ \ I2r = \sqrt{\frac{Pcu2}{3R2}} = \sqrt{\frac{170}{3 \times 0.8}} = 8.4A$$

$$\mu s = \frac{Ts}{Tr} = \left(\frac{Isc}{I2r}\right)^2 Sr = \left(\frac{60}{8.4}\right)^2 0.03 = 1.2$$

$$\mu m = \frac{Tm}{Tr} = \frac{sm^2 + sr^2}{2sr\ sm} = \frac{0.2^2 + 0.03^2}{2*0.2*0.03} = 3.4$$

$$\frac{T_S}{T_M} = \frac{T_S}{T_r} X \frac{T_r}{T_M} = 0.35$$

Or $\mu sm = \frac{2S_m}{1 + {S_m}^2}$

Ex2) The synchronous speed of 4-pole,50HZ,3-phase IM drops by 4% at full load ,its rotor standstill impedance is 0.2+j0.8 .find the full load output power and Ts if Tm=100 N-M

Sol:-

$$n1 = \frac{60f1}{p} = 1500\text{rpm} \quad \text{Sr} = 0.04 \qquad N\text{r} = 1440\text{rpm}$$

$$Sm = \frac{R2}{X2} = \frac{0.2}{0.8} = 0.25$$

$$\mu m = \frac{Tm}{Tr} = \frac{Sm^2 + Sr^2}{2Sm\,Sr} = \frac{0.25^2 + 0.04^2}{2 \times 0.25 \times 0.04} = 3.205$$

$$Tr = \frac{Tm}{\mu m} = \frac{100}{3.205} = 31.2 \;\; N.M$$

$$P2 = \frac{2\pi n}{60} Tr = 2\pi \, \frac{1440}{60} .31.2 = 4700\omega$$

$$\mu sm = \frac{Ts}{Tm} = \frac{2\,sm}{1 + sm^2} = \frac{2 \times \, 0.25}{1 + 0.25^2} = 0.47$$

$$Ts = \mu sm\,Tm = 0.47 \times 100 = 47 \; N.M$$

Ex3) A 3-phase , 4-pole,1 5kw ,50HZ,1455 rpm Im.its rated slip is 3%and stand still rotor impedance is (0.2+j0.8) find the max. torque and the speed at which occurs.

Sol:-

$$n1 = \frac{60F1}{P} = 1500\text{rpm}, n = (1 - 0.03)1500 = 1455rpm$$

$$Sm = \frac{R2}{X2} = \frac{0.2}{0.8} = 0.25$$

$$Tr = \frac{P2}{\omega} = \frac{60 \times 15000}{2\pi \times 1455} = 98.5 \; N.M$$

$$\mu m = \frac{Tm}{Tr} = \frac{Sm^2 + Sr^2}{2Sm\,Sr} = \frac{0.25^2 + 0.03^2}{2 \times 0.25 \times 0.03} = 4.77$$

$$Tm = Tr \times \mu m = 98.5 \times 4.77 = 470 \; N.M$$

The speed at which the max. Torque occurs

$$nm = (1 - sm)n1 = (1 - 0.25)1500 = 1125rpm$$

Ex4) A 3-phase ,SRIM having stand still rotor impedance is (0.2 +j0.6) Ω/phase . find the value of the additional resistance to be added to the rotor winding in order to increase the starting torque to ↑75% of the max. torque.

Sol:-

$$\frac{Ts}{Tm} = \frac{2sm}{1 + sm^2} = 0.75$$

$$0.75sm^2 - 2sm + 0.75 = 0$$

$$sm = \frac{2 \pm \sqrt{2^2 - 4 \times 0.75 \times 0.75}}{2 \times 0.75} = 0.45$$

$$sm = \frac{Rt}{X2} = \frac{R2 + R_{add}}{X2} = 0.45$$

$$R_{add} = 0.6 \times 0.45 - 0.2 = 0.07\Omega$$

4- Starting Methods for Induction Motors

A 3-phase induction motor is theoretically self starting. The stator of an induction motor consists of 3-phase windings, which when connected to a 3-phase supply creates a rotating magnetic field. This will link and cut the rotor conductors which in turn will induce a current in the rotor conductors and create a rotor magnetic field. The magnetic field created by the rotor will interact with the rotating magnetic field in the stator and produce rotation.
Therefore, 3-phase induction motors employ a starting method not to provide a starting torque at the rotor, but because of the following reasons;
1) Reduce heavy starting currents and prevent motor from overheating.
2) Provide overload and no-voltage protection.
There are many methods in use to start 3-phase induction motors. Some of the common methods are;

1. Direct On-Line Starter (DOL)
2. Star-Delta Starter
3. Auto Transformer Starter
4. Rotor Impedance Starter
5. Power Electronics Starter

4-1 Direct On-Line Starter (DOL)

The Direct On-Line (DOL) starter is the simplest and the most inexpensive of all starting methods and is usually used for squirrel cage induction motors. It directly connects the Contacts of the motor to the full supply voltage. The starting current is very large, normally 6 to 8 times the rated current. The starting torque is likely to be 0.75 to 2 times the full load Torque. In order to avoid excessive voltage drops in the supply line due to high starting currents, the DOL starter is used only for motors with a rating of less than 5KW

There are safety mechanisms inside the DOL starter which provides protection to the motor as well as the operator of the motor. The power and control circuits of induction motor with DOL starter are shown in figure(1).

* K1M Main contactor

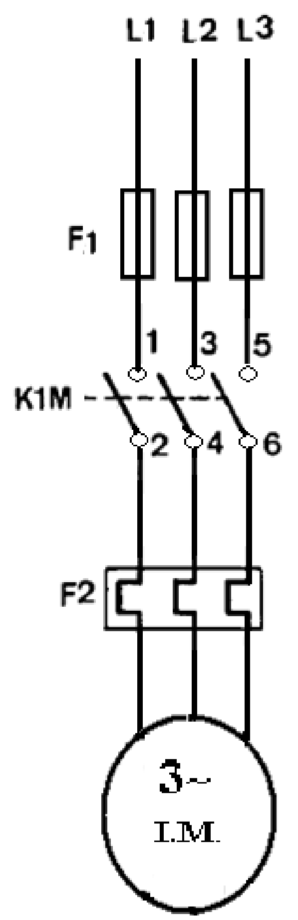


Fig.(1): power and control circuits of I.M. with DOL starter

4-2 Star-Delta Starter

The star delta starting is a very common type of starter and extensively used, compared to the other types of the starters.

This method used reduced supply voltage in starting. Figure(2) shows the connection of a 3phase induction motor with a star – delta starter.

The method achieved low starting current by first connecting the stator winding in star configuration, and then after the motor reaches a certain speed, throw switch changes the winding arrangements from star to delta configuration. By connecting the stator windings, first in star and then in delta, the line current drawn by the motor at starting is reduced to one-third as compared to starting current with the windings connected in delta. At the time of starting when the stator windings are start connected, each stator phase gets voltage $\frac{Vl}{\sqrt{3}}$, where Vl□ is the line voltage. Since the torque developed by an induction motor is proportional to the square of the applied voltage, star- delta starting reduced the starting torque to one – third that obtainable by direct delta starting.

- K2M Main Contactor
- K3M Delta Contactor
- K1M Star Contactor
- F1 Thermal Overload Relay

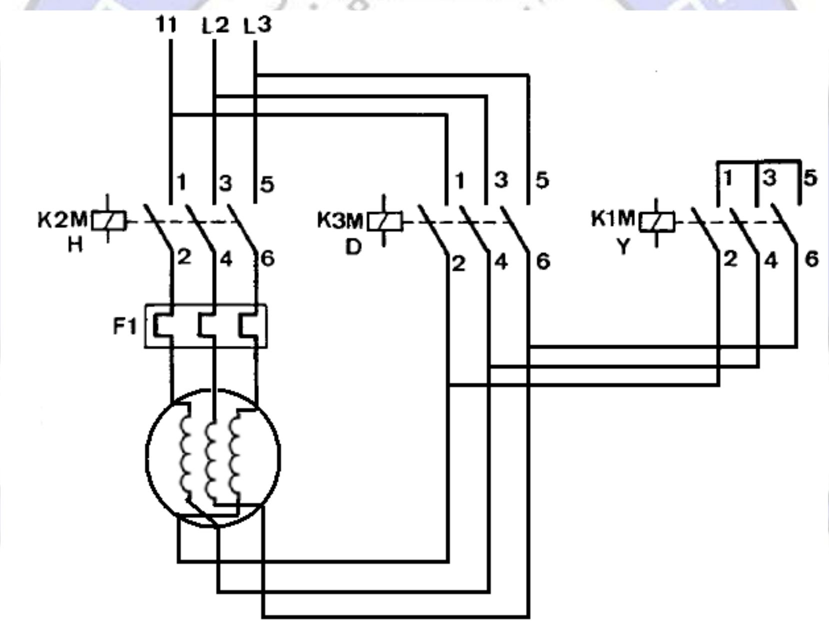


Fig.(2) Induction Motor with Star Delta Starter

4-3 Auto Transformer Starter

The operation principle of auto transformer method is similar to the star delta starter method. The starting current is limited by (using a three phase auto transformer) reduce the initial stator applied voltage. The auto transformer starter is more expensive, more complicated in operation and bulkier in construction when compared with the star – delta starter method. But an auto transformer starter is suitable for both star and delta connected motors, and the starting current and torque can be adjusted to a desired value by taking the correct tapping from the auto transformer. When the star delta method is considered, voltage can be adjusted only by factor of .$^{1}/_{\sqrt{3}}$ Figure (3) shows the connection of a 3phase induction motor with auto transformer starter.

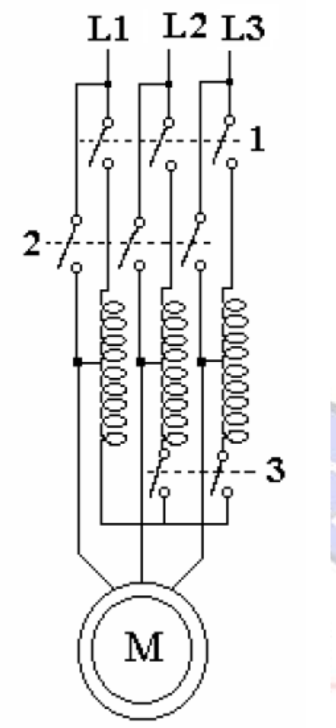


Fig.(3) shows I.M with auto transformer starter.

4-4 Rotor Impedance Starter

This method allows external resistance to be connected to the rotor through slip rings and brushes. Initially, the rotor resistance is set to maximum and is then gradually decreased as the motor speed increases, until it becomes zero.

The rotor impedance starting mechanism is usually very bulky and expensive when compared with other methods. It also has very high maintenance costs. Also, a considerable amount of heat is generated through the resistors when current runs through them. The starting frequency is also limited in this method. However, the rotor impedance method allows the motor to be started while on load. Figure (4) shows the connection of a 3phase induction motor with rotor resistance starter.

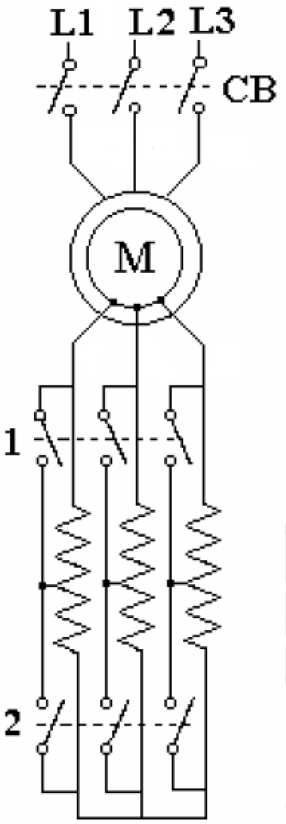


Fig. (4) Shows the I.M. with rotor resistance starter.

Example (9):

It is desired to install a 3-phase cage induction motor restricting the maximum line current drawn from a 400 V 3-phase supply to 120 A. if the starting current is 6 times full load current, what is the maximum permissible full load kVA of the motor when

i. It is directly connected to the mains

ii. It is connected through an auto-transformer with a tapping of 60%

iii. It is designed for used with star-delta starter.

Solution:

i. Direct-on-line starting

Maximum line current, $I_L = 120A$

Starting current $I_{st} = 6 \times \text{full load current} = 6I_{fl}$

Since the maximum line current drawn from the supply is 120A

$$6I_{ft} = 120, \qquad I_f = \frac{120}{6} = 20A$$

Maximum permissible rating of the motor

$$= \sqrt{3}V_L I_{ft} = \sqrt{3} \times 20 \times 400 = 13856 \text{ VA} = 13.856 \text{ k VA}$$

ii. Auto-transformer starting

$$I_{st} = x^2 I_{sc} = x^2(6I_{ft})$$

$$120 = (0.6)^2(6I_{ft})$$

$$I_{ft} = \frac{120}{6 \times (0.6)^2} = 55.55A$$

Maximum permissible rating of the motor

$$= \sqrt{3}V_L I_{ft} = \sqrt{3} \times 400 \times 55.55 = 38.49\text{ k VA}$$

iii. Star-delta starting

$$I_{st} = \frac{1}{3}(6I_{ft})$$

$$120 = 2I_{ft}, \qquad I_{ft} = 60A$$

Maximum permissible kVA rating of the motor

$$= \sqrt{3}V_L I_{ft} = \sqrt{3} \times 400 \times 60 = 41.56\text{ k VA}$$

5- Speed Control of Induction Motor

Induction motors are not good machines for applications requiring considerable speed control. The normal operating range of a typical induction motor is confined to less than 5% slip, and the speed variation is more or less proportional to the load. if slip is made higher, rotor copper losses will be high as well.

There are basically 2 general methods to control induction motor's speed:

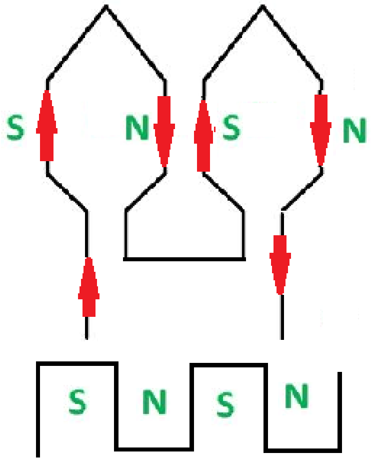
a) Varying stator and rotor magnetic field speed

b) Varying slip

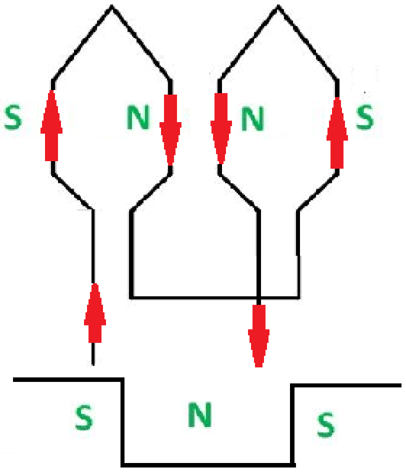
Varying the magnetic field speed may be achieved by varying the **electrical frequency** or by

5-1 Changing the Number of Poles.

Varying slip may be achieved by **varying rotor resistance** or **varying the terminal voltage**. In the method of consequent poles, a single stator winding is divided into few coil groups. The terminals of all these groups are brought out. By simply changing the coil connections, the number of poles can be changed. In practice, the stator windings are divided only in two coil groups. The number of poles can be changed in the ratio of 2:1



4 Pole



2 Pole

Induction Motor Speed Control by Pole Changing

There are 2 approaches possible:

a) Method of Consequent Poles (Old Method)

b) Multiple Stator Windings Method

Method of Consequent Poles

General Idea:

Consider one phase winding in a stator. By changing the current flow in one portion of the stator windings as such that it is similar to the current flow in the opposite portion of the stator will automatically generate an extra pair of poles.

By applying this method, the number of poles may be maintained (no changes), doubled or halfed , hence would vary its operating speed.

In terms of torque, the maximum torque magnitude would generally be maintained.

Disadvantage:

This method will enable speed changes in terms of 2:1 ratio steps, hence to obtained variations in speed, multiple stator windings has to be applied. Multiple stator windings have extra sets of windings that may be switched in or out to obtain the required number of poles. Unfortunately this would an expensive alternative.

5-2 Speed Control by Changing the Line Frequency

Changing the electrical frequency will change the synchronous speed of the machine.

Changing the electrical frequency would also require an adjustment to the terminal voltage in order to maintain the same amount of flux level in the machine core. If not the machine will experience:

a) Core saturation (non linearity effects)
b) Excessive magnetization current.

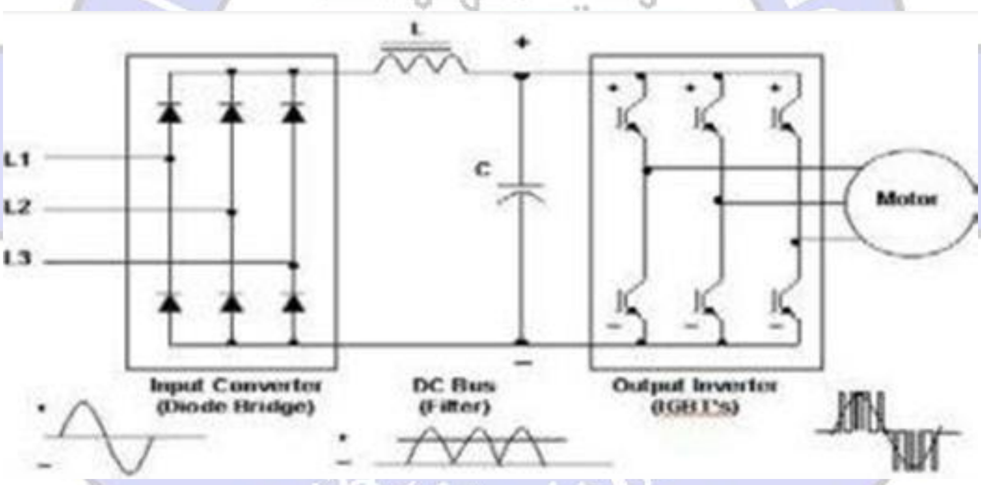
Varying frequency with or without adjustment to the terminal voltage may give 2 different effects:

a) Vary frequency, stator voltage adjusted – generally vary speed and maintain operating torque.
b) Vary Frequency, stator voltage maintained – able to achieve higher speeds but a reduction of torque as speed is increased.

There may also be instances where both characteristics are needed in the motor operation; hence it may be combined to give both effects.

With the arrival of solid-state devices/power electronics, line frequency change is easy to achieved and it is more versatile to a variety of machines and application

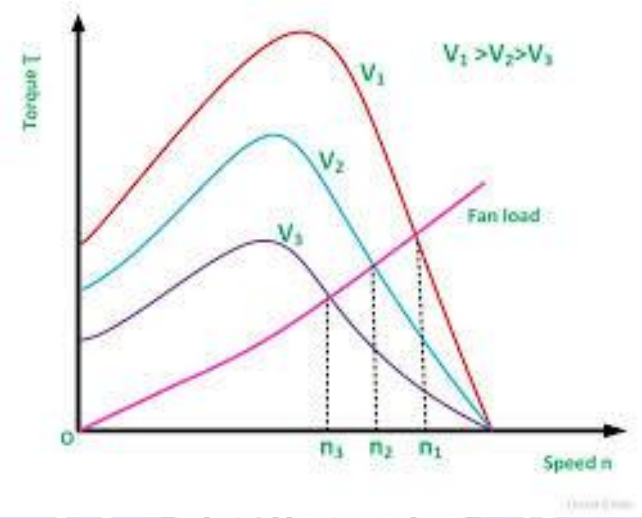
.

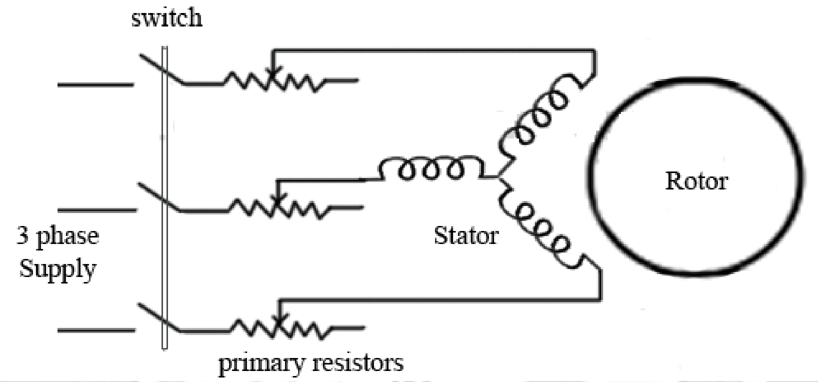


5-3 Speed Control by Changing the Line Voltage

From the torque equation of the induction machine given , we can see that the torque depends on the square of the applied voltage. The variation of speed torque curves with respect to the applied voltage is shown in fig. below. These curves show that the slip at maximum torque ˆs remains same, while the value of stall torque comes down with decrease in applied voltage. The speed range for stable operation remains the same. Further, we also note that the starting torque is also lower at lower voltages. Thus, even if a given voltage level is sufficient for achieving the running torque, the machine may not start. This method of trying to control the speed is best suited for loads that require very little starting torque, but their torque requirement may increase with speed.

Varying the terminal voltage will vary the operating speed but with also a variation of operating torque. In terms of the range of speed variations, it is not significant hence this method is only suitable for small motors only.





5-4 Speed Control by Changing the Rotor Resistance

It is only possible for wound rotor applications but with a cost of reduced motor efficiency.

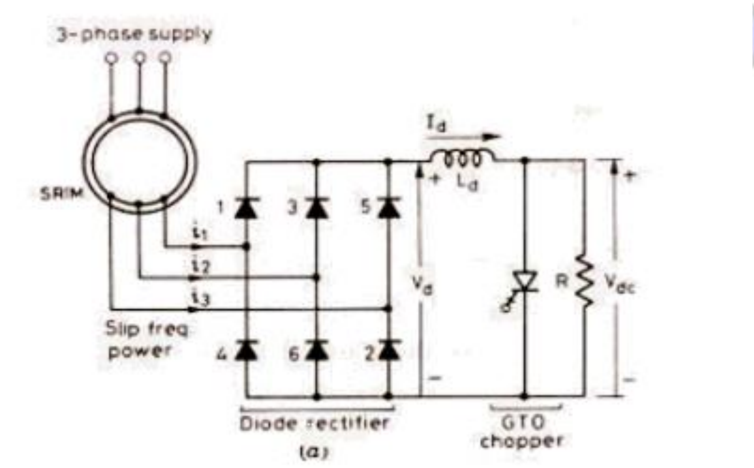
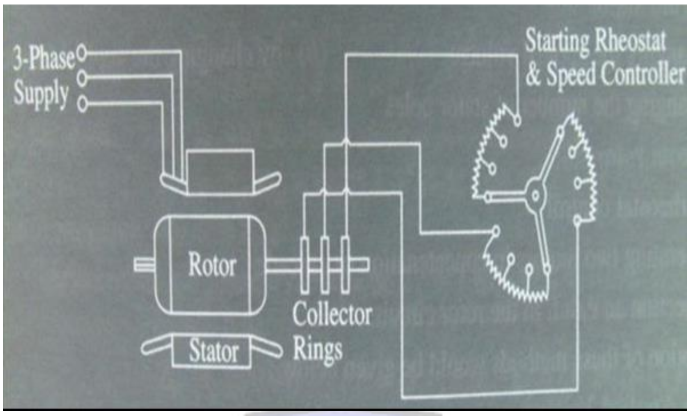
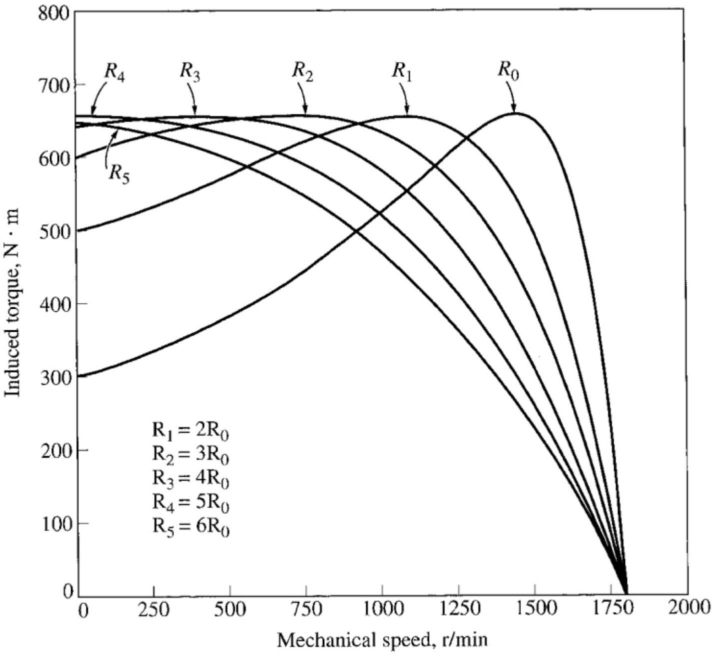


Fig. (a) SRIM control by static variation of external rotor resistance





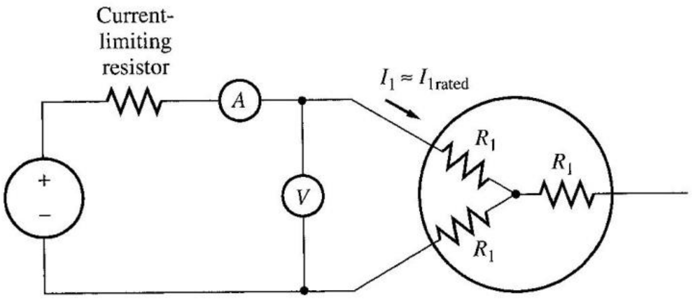
6- The DC Test

This is a test for R1 independent of R2, X1, and X2.

DC voltage is applied to the terminals of the stator windings of the induction motor. Since it is DC supply, f = 0, hence no induced current in the rotor circuit. Current will flow through the stator circuit.

Reactance is zero at dc. Thus, the only quantity limiting current flow in the motor is the stator resistance, and it can be determined.

Assume we have a Y connected induction motor circuit as shown:



a) DC voltage is applied across the motor terminal and current flow is adjusted to rated condition
(to simulate normal operating condition)
b) Voltage and current flow is noted.
Based upon the test configuration,

$$2R_1 = \frac{V_{DC}}{I_{DC}} \quad , \quad \therefore R_1 = \frac{V_{DC}}{2I_{DC}}$$

Since we are able to determine the value of R1, hence PSCL can be calculated. Unfortunately, this method
is not accurate since it is done using a DC power source where skin effects, that occurs when an ac
voltage is applied to the windings, are neglected.

Single-Phase Motors

1 Introduction

Single phase motors are the most familiar of all electric motors because they are extensively used in home appliances, shops, offices etc. It is true that single phase motors are less efficient substitute for 3-phase motors but 3-phase power is normally not available except in large commercial and industrial establishments. Since electric power was originally generated and distributed for lighting only, millions of homes were given single-phase supply. This led to the development of single-phase motors. Even where 3-phase mains are present, the single-phase supply may be obtained by using one of the three lines and the neutral. Single-phase induction motors are usually two-pole or four-pole, rated at 2 hp or less, while slower and larger motor can be manufactured for special purposes. They are widely used in domestic appliances and for a very large number of low power drives in industry. The single phase induction motor resembles, three-phase, squirrel-cage motor except that, single phase induction motor has no starting torque and some special arrangement have to be made to make it as self starting. We shall focus our attention on the construction, working and characteristics of commonly used single-phase motors.

2 Types of Single-Phase Motors

Single-phase motors are generally built in the fractional-horsepower range and may be classified into the following four basic types:

1. Single-phase induction motors

(i) split-phase type (ii) capacitor start type (iii) capacitor start capacitor run type (v) shaded-pole type

2. A.C. series motor or universal motor

3. Repulsion motors

(i) Repulsion-start induction-run motor (ii) Repulsion-induction motor

4. Synchronous motors

(i) Reluctance motor (ii) Hysteresis motor

3 Single-Phase Induction Motors

A single phase induction motor is very similar to a 3-phase squirrel cage induction motor. Unlike a 3-phase induction motor, a single-phase induction motor is not self starting but requires some starting means. The single-phase stator winding produces a magnetic field that pulsates in strength in a sinusoidal manner. The field polarity reverses after each half cycle but the field does not rotate. Consequently, the alternating flux cannot produce rotation in a stationary squirrel-cage rotor. However, if the rotor of a single-phase motor is rotated in one direction by some mechanical means, it will continue to run in the direction of rotation. As a matter of fact, the rotor quickly accelerates until it reaches a speed slightly below the synchronous speed. Once the motor is running at this speed, it will continue to rotate even though single-phase current is flowing through the stator winding. This method of starting is generally not convenient for large motors. Figure 1 shows picture of single phase induction motor.

Figure 1 Single phase induction motor.

4 Construction of single phase induction motor

The construction parts on of single phase induction motor consist of main two parts: stationary stator and revolving rotor. The stator separate from rotor by small air gap have ranges from 0.4 mm to 4 mm depends to size of motor.

4.1 Stator

The single-phase motor stator has a laminated iron core with two windings arranged perpendicularly, One is the main and the other is the auxiliary winding or starting winding as showing in the figure 2. It consists of a steel frame which encloses a hollow, cylindrical core made up of thin laminations of silicon steel to reduce hysteresis and eddy current losses. A number of evenly spaced slots are provided on the inner periphery of the laminations.

Figure 2 Stator of single phase induction motor.

4.2 Rotor

The rotor, mounted on a shaft, is a hollow laminated core having slots on its outer periphery. The winding placed in these slots (called rotor winding) may be one of the following two types:

(i) Squirrel cage rotor:

It consists of a laminated cylindrical core having parallel slots on its outer periphery. One copper or aluminum bar is placed in each slot. All these bars are joined at each end by metal rings called end rings [See Fig. 3]. This forms a permanently short-circuited winding which is indestructible. The entire construction (bars and end rings) resembles a squirrel cage and hence the name. The rotor is not connected electrically to the supply but has current induced in it by transformer action from the stator. Those induction motors which employ squirrel cage rotor are called squirrel cage induction motors. Most of single phase induction motors use squirrel cage rotor as it has a remarkably simple and robust construction enabling it to operate in the most adverse circumstances. However, it suffers from the disadvantage of a low starting torque. It is because the rotor bars are permanently short-circuited and it is not possible to add any external resistance to the rotor circuit to have a large starting torque. In this type of rotor the bars conductor are skew to reduce the noise.

Figure 3 Squirrel cage rotor.

(ii) Wound Rotor:

It consists of a laminated cylindrical core and carries a single-phase winding, similar to the one on the stator. The open ends of the rotor winding are brought out and joined to three insulated slip rings mounted on the rotor shaft with one brush resting on each slip ring. The two brushes are connected to a single-phase star-connected rheostat as shown in Figure 4. At starting, the external resistances are included in the rotor circuit to give a large starting torque. These resistances are gradually reduced to zero as the motor runs up to speed. The external resistances are used during starting period only. When the motor attains normal speed, the two brushes are short-circuited so that the wound rotor runs like a squirrel cage rotor.

Figure 4 wound rotor of single phase induction motor.

5 . Applications:

Single phase induction motors are in very wide use in industry especially in fractional horse-power field. They are extensively used for electrical drive for low power constant speed apparatus such as machine tools, domestic apparatus and agricultural machinery in circumstances where a three-phase supply is not readily available.

Single phase induction motors sizes vary from 1/400 kw to 1/25 kw are used in toys, hair dryers, vending machines etc.

Universal motor is widely used in portable tools, vacuum cleaners& kitchen equipment.

Disadvantages:

Though these machines are useful for small outputs, they are not used for large powers as they suffer from many disadvantages and are never used in cases where three-phase machines can be adopted. The main disadvantages of single-phase induction motors are:

1. Their output is only 50% of the three-phase motor, for a given frame size and temperature rise.
2. They have lower power factor.
3. Lower efficiency
4. These motors do not have inherent starting torque
5. More expensive than three-phase motors of the same output.
6. Low overload capacity

6 Principle of Work

A single-phase induction motor is not self starting but requires some starting means. The single-phase stator winding produces a magnetic field that pulsates in strength in a sinusoidal manner. The field polarity reverses after each half cycle but the field does not rotate. Consequently, the alternating flux cannot produce rotation in a stationary squirrel-cage rotor. However, if the rotor of a single-phase motor is rotated in one direction by some mechanical means, it will continue to run in the direction of rotation. As a matter of fact, the rotor quickly accelerates until it reaches a speed slightly below the synchronous speed. Once the motor is running at this speed, it will continue to rotate even though single-phase current is flowing through the stator winding. This method of starting is generally not convenient for large motors. Figure 5 shows single-phase induction motor having a squirrel cage rotor and a single phase distributed stator winding. Such a motor inherently docs not develop any starting torque and, therefore, will not start to rotate if the stator winding is connected to single-phase A.C.

supply. However, if the rotor is started by auxiliary means, the motor will quickly attain final speed. This strange behavior of single-phase induction motor can be explained on the basis of double-field revolving theory.

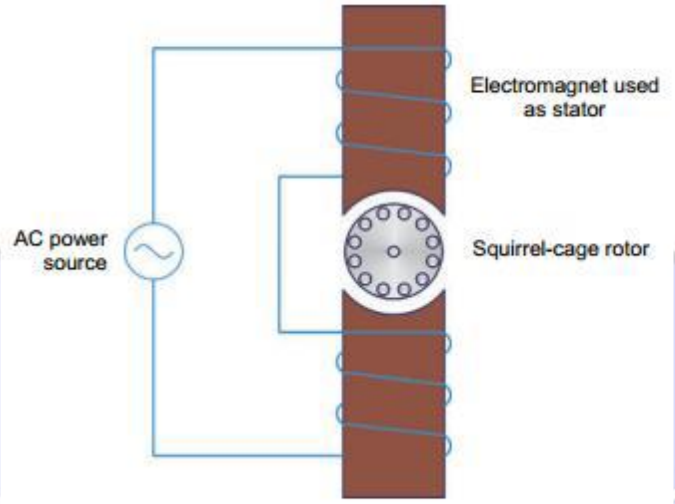


Figure 5 single-phase induction motor.

6.1 Operation of Single phase induction motor

(i) When stator winding is energized from a.c. supply, a rotating magnetic field (RMF) is set up which rotates round the stator at synchronous speed Ns (= 60 f/P), when f = frequency and P No. of poles pairs.

(ii) The rotating field passes through the air gap and cuts the rotor conductors, which as yet, are stationary. Due to the relative speed between the rotating flux and the stationary rotor, electrical motive forces (EMF) are induced in the rotor conductors. Since the rotor circuit is short-circuited, currents start flowing in the rotor conductors (Figure 6). The flux from the stator will cut the coil in the rotor and since the rotor coils are short circuited, according to Faraday’s law of electromagnetic induction, current will start flowing in the coil of the rotor.

(iii) The current-carrying rotor conductors are placed in the magnetic field produced by the stator. Consequently, mechanical force acts on the rotor conductors. The sum of the mechanical forces on all the rotor conductors produces a torque which tends to move the rotor in the same direction as the rotating field with speed N =Ns (1-S) when S= slip and N = rotor speed (Figure 6).

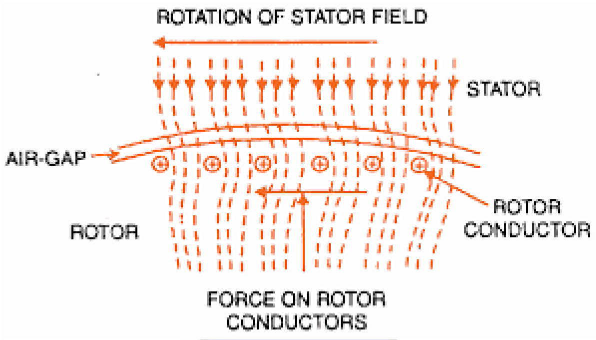


Figure 6 Transmission of Rotate magnetic field

6.2 Cross-field theory

The principle of operation of a single-phase induction motor can be explained from the cross-field theory. As soon as the rotor begins to turn, a speed emf E is induced in the rotor conductors, as they cut the stator flux Fs. This voltage increases as the rotor speed increases. It causes current Ir to flow in the rotor bars facing the stator poles as shown in figure 7.7 . These currents produce an ac flux FR which act at right angle to the stator flux Fs. Equally important is the fact that FR does not reach its maximum value at the same time as FS does, in effect, FR lags almost 90o behind FS, owing to the inductance of the rotor The combined action of Fs and FR produces a revolving magnetic field, similar to that in a three-phase motor. The value of FR increases with increasing speed, becoming almost equal to Fs at synchronous speed. The flux rotates counterclockwise in the same direction as the rotor and it rotates at synchronous speed irrespective of the actual speed of the rotor. As the motor approaches synchronous speed, FR becomes almost equal to Fs and a nearly perfect revolving field is produces.

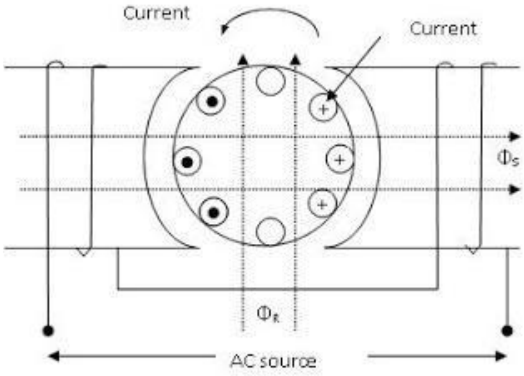


Figure 7 Current induced in the rotor bars due to rotation.

6.3 Double-field revolving theory

When the stator winding (distributed one as stated earlier) carries a sinusoidal current (Being fed from a single-phase supply), a sinusoidal space distributed mmf, whose peak or maximum value pulsates (alternates) with time, is produced in the air gap. This sinusoidal varying flux (φ) is the sum of two rotating fluxes or fields, the magnitude of which is equal to half the value of the alternating flux (φ / 2), and both the fluxes rotating synchronously at the speed, in opposite directions. The first set of figures (Figure 8a (i-iv)) show the resultant sum of the two rotating fluxes or fields, as the time axis (angle) is changing from θ = 0° to π °(180) . Figure 8b shows the alternating or pulsating flux (resultant) varying with time or angle.

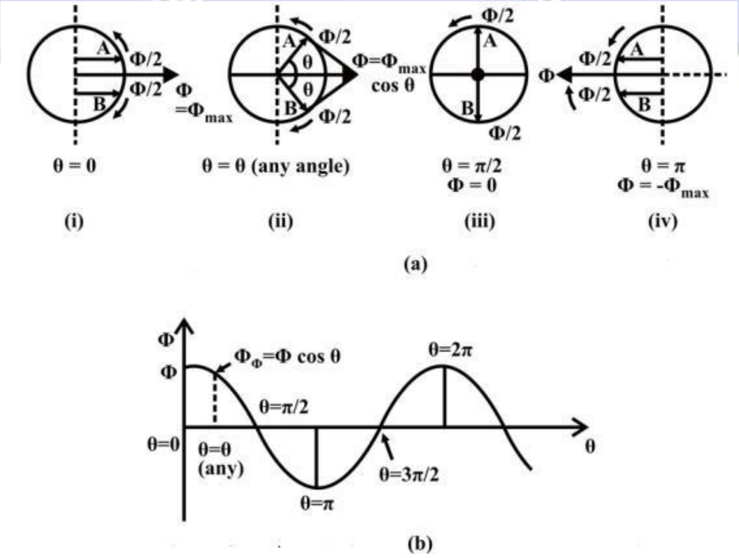


Figure 8 Double field revolving

The flux or field rotating at synchronous speed, say, in the anticlockwise direction, i.e. the same direction, as that of the motor (rotor) taken as positive induces EMF (voltage) in the rotor conductors. The rotor is a squirrel cage one, with bars short circuited via end rings. The current flows in the rotor conductors, and the electromagnetic torque is produced in the same direction as given above, which is termed as positive (+ve). The other part of flux or field rotates at the same speed in the opposite (clockwise) direction, taken as negative. So, the torque produced by this field is negative (-ve), as it is in the clockwise direction, same as that of the direction of rotation of this field. Two torques are in the opposite direction, and the resultant (total) torque is the difference of the two torques produced. Let the flux φ1 rotate in anti clockwise direction and flux φ2 in clockwise direction. The flux φ1 will result in the production of torque T1 in the anti clockwise direction and flux φ2 will result in the production of torque T2 In the clockwise direction. Thus the point of zero slip for one field corresponds to 200% slip for the other as explained later. The value of 100% slip (standstill condition) is the same for both the fields. This fact is illustrated in Figure 7.9. At standstill, these two torques are equal and opposite and the net torque developed is zero. Therefore, single-phase induction motor is not self-starting. Note that each rotating field tends to drive the rotor in the direction in which the field rotates.

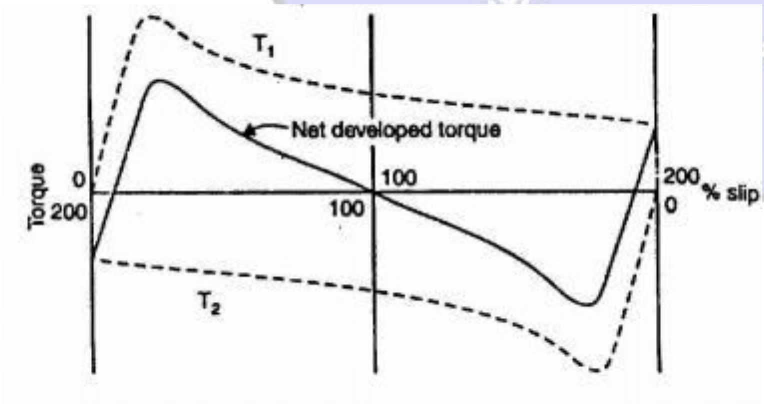


Figure 7.9 Speed Torque characteristics.

Now assume that the rotor is started by spinning the rotor or by using auxiliary circuit, in say clockwise direction. The flux rotating in the clockwise direction is the forward rotating flux (φf) and that in the other direction is the backward rotating flux (φb). The slip w.r.t. the forward flux will be

$$Sf = \frac{Ns - N}{Ns} = 1 - \frac{N}{Ns} \; or \; \frac{N}{Ns} = 1 - S$$

The rotor rotates opposite to the rotation of the backward flux. Therefore, the slip w.r.t. the backward flux will be

$$Sb = \frac{Ns - (-N)}{Ns} = \frac{Ns + N}{Ns} = 1 + \frac{N}{Ns} = 1 + (1 - S) = 2 - S$$

Thus fur forward rotating flux, slip is s (less than unity) and for backward rotating flux, the slip is 2 − s (greater than unity). Since for usual rotor resistance/reactance ratios, the torques at slips of less than unity arc greater than those at slips of more than unity, the resultant torque will be in the direction of the rotation of the forward flux. Thus if the motor is once started, it will develop net torque in the direction in which it has been started and will function as a motor.

7 Making Single-Phase Induction Motor Self-Starting

The single-phase induction motor is not self starting and it is undesirable to resort to mechanical spinning of the shaft or pulling a belt to start it. To make a single-phase induction motor self-starting, we should somehow produce a revolving stator magnetic field. This may be achieved by converting a single-phase supply into two-phase supply through the use of an additional winding. When the motor attains sufficient speed, the starting means (i.e., additional winding) may be removed depending upon the type of the motor. As a matter of fact, single-phase induction motors are classified and named according to the method employed to make them self-starting.

(i) Split-phase motors-started by two phase motor action through the use of an auxiliary or starting winding.

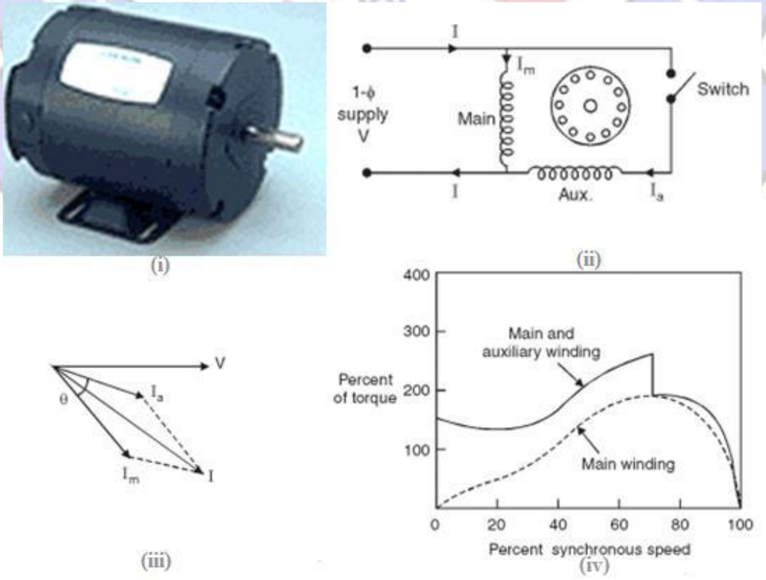
(ii) Capacitor start motors-started by two-phase motor action through the use of an auxiliary winding and a capacitor.

(iii) Capacitor start Capacitor run motors-started by two-phase motor action through the use of an auxiliary winding and two capacitors.

(v) Shaded-pole motors-started by the motion of the magnetic field produced by means of a shading coil around a portion of the pole structure.

7.1 Split-phase induction motors

The stator of a split-phase induction motor is provided with an auxiliary or starting winding S in addition to the main or running winding M. The starting winding is located 90° electrical from the main winding and the picture of split phase induction motor [See Fig12 (i))] and operates only during the brief period when the motor starts up. The two windings are so resigned that the starting winding S has a high resistance and relatively small reactance while the main winding M has relatively low resistance and large reactance to be as inductance (the current delay with voltage) to make shifting current as shown in the schematic connections in Figure 12 (ii)). Consequently, the currents flowing in the two windings have reasonable phase difference c (25° to 30°) as shown in the pharos diagram this shifting in current its necessary for starting torque in Figure 12 (iii)). Figure 12 (iv) shows typical torque speed characteristics.



(i) (ii) (iii) (iv)

Figure 12 Split-phase induction motors.

7.1.1 Operation

(i) When the two stator windings are energized from a single-phase supply, the main winding carries current Im while the starting winding carries current Is.

(ii) Since main winding is made highly inductive while the starting winding highly resistive, the currents Im and Is have a reasonable phase angle a (25° to 30°) between them. Consequently, a weak revolving field approximating to that of a 2-phase machine is produced which starts the motor.

(iii) When the motor reaches about 80% of synchronous speed, the centrifugal switch opens the circuit of the starting winding. The motor then operates as a single-phase induction motor and continues to accelerate till it reaches the normal speed. The normal speed of the motor is below the synchronous speed and depends upon the load on the motor.

7.1.2 Characteristics

(i) The sinning torque is 2 times the full-loud torque mid (lie starting current is 6 to 8 times the full-load current. (ii) Due to their low cost, split-phase induction motors are most popular single phase motors in the market.

(iii) Since the starting winding is made of fine wire, the current density is high and the winding heats up quickly. If the starting period exceeds 5 seconds, the winding may burn out unless the motor is protected by built-in-thermal relay. This motor is, therefore, suitable where starting periods are not frequent.

(iv) An important characteristic of these motors is that they are essentially constant-speed motors. The speed variation is 2-5% from no-load to full load.

(v) These motors are suitable where a moderate starting torque is required and where starting periods are infrequent e.g., to drive:

(a) fans (b) washing machines (c) oil burners (d) small machine tools etc.

The power rating of such motors generally lies between 60 W and 250 W.

7.2 Capacitor induction Motor

The capacitor-start motor is identical to a split-phase motor except that the starting winding has as many turns as the main winding. The picture of capacitor start induction motor is shows in Figure 13 (i). Moreover, a capacitor C (3-20 μF) is connected in series with the starting winding as shown in Figure 13 (ii)). The value of capacitor is so chosen that Is leads Im by about 80° which is considerably greater than 25° found in split-phase motor [See Figure 13 (iii))]. Figure 13(iv) shows typical torque speed characteristic.

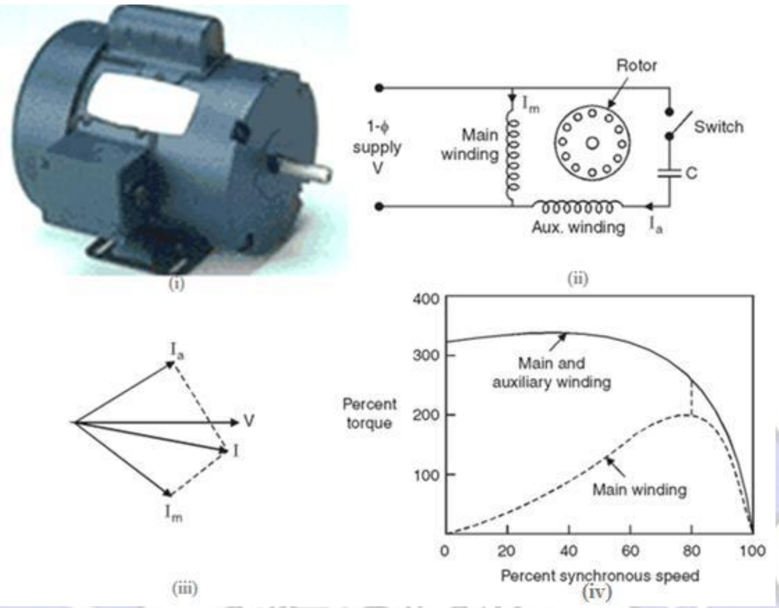


Fig. 13 Capacitor-Start Motor.

7.2.1 Operation

(i) When the two stator windings are energized from a single-phase supply, the main winding carries current Im while the starting winding carries current Is.

(ii) Due to cap acitance the currents Im and Is have a reasonable phase angle a (80°) between them.

(iii) When starting torque is much more than that of a split-phase motor Again, the starting winding is opened by the centrifugal switch when the motor attains about 80% of synchronous speed. The motor then operates as a single-phase induction motor and continues to accelerate till it reaches the normal speed.

7.2.2 Characteristics

(i) Although starting characteristics of a capacitor-start motor are better than those of a split-phase motor, both machines possess the same running characteristics because the main windings are identical.

(ii) The phase angle between the two currents is about 80° compared to about 25° in a split-phase motor. Consequently, for the same starting torque, the current in the starting winding is only about half that in a split-phase motor. Therefore, the starting winding of a capacitor start motor heats up less quickly and is well suited to applications involving either frequent or prolonged starting periods.

(iii) Capacitor-start motors are used where high starting torque is required and where the starting period may be long e.g., to drive:

(a) compressors (b) large fans (c) pumps (d) high inertia loads
The power rating of such motors lies between 120 W and 7-5 kW.

7.3 Capacitor start Capacitor run induction motors

This motor is identical to a capacitor-start motor except that starting winding is not opened after starting so that both the windings remain connected to the supply when running as well as at starting. Two designs are generally used. Figure 14 (i) shows picture of capacitor start capacitor run induction motor. This design eliminates the need of a centrifugal switch and at the same time improves the power factor and efficiency of the motor. In the other design, two capacitors C1 and C2 are used in the starting winding as shown in Figure 14 (ii). The value of capacitor is so chosen that Is leads Im by about 80° [See Figure 14 (iii))]. The smaller capacitor C1 required for optimum running conditions is permanently connected in series with the starting winding. The much larger capacitor C2 is connected in parallel with C1 for optimum starting and remains in the circuit during starting. The starting capacitor C2 is disconnected when the motor approaches about 80% of synchronous speed. The motor then runs as two-phase induction motor. Figure 14 (iv) shows typical torque speed characteristic.

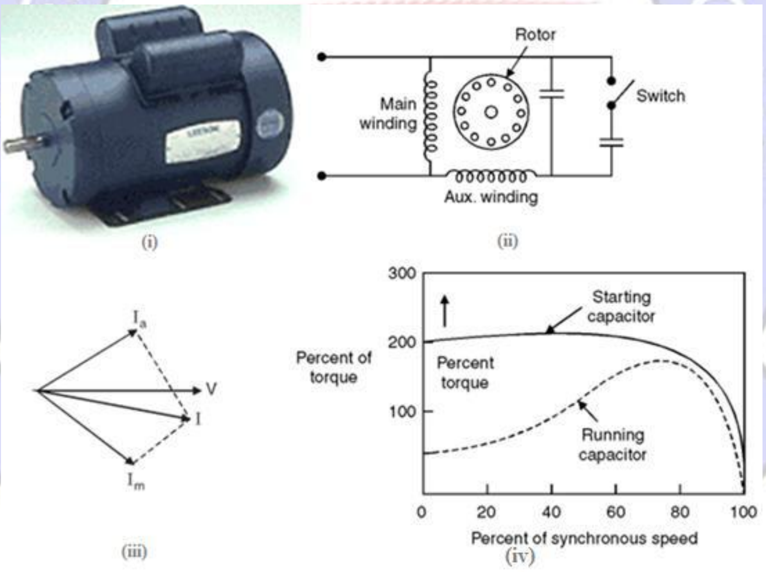


Fig. 14 Capacitor start Capacitor run induction motors.

7.3.1 Operation

(i) When the two stator windings are energized from a single-phase supply, the main winding carries current IM while the starting winding carries current Is.

(ii) Due to capacitance C1 the currents Im and Is have a reasonable phase angle a (80°) between them.

(iii) When The starting capacitor C2 is disconnected when the motor approaches about 80% of synchronous speed. The motor then runs as two-phase induction motor.

7.3.2 Characteristics

(i) The starting winding and the capacitor can be designed for perfect 2-phase operation at any load. The motor then produces a constant torque and not a pulsating torque as in other single-phase motors.

(ii) Because of constant torque, the motor is vibration free and can be used in: (a) hospitals (b) studios and (c) other places where silence is important.

7.4 Shaded-pole induction motors

A picture of shaded pole induction motor are shows in Figure 15 (i). A typical shaded-pole motor with a cage rotor is shown in Figure 15 (ii). This is a single phase induction motor, with main winding in the stator. A small portion of each pole is covered with a short-circuited, single-turn copper coil called the shading coil. The sinusoidal varying flux created by ac (single-phase) excitation of the main winding induces in the shading coil. As a result, induced currents flow in the shading coil producing their own flux in the shaded portion of the pole. as shown in Figure 15 (iii) and lags the flux φ_m' of the remaining pole by the angle α . The two sinusoidal varying fluxes φ_m' and φ_{sp}' are displaced in space as well as have a time phase difference (α), thereby producing forward and backward rotating fields, which produce a net torque. It may be noted that the motor is self-starting unlike a single-phase single-winding motor. It is seen from the phasor diagram (Figure 15 (iii) that the net flux in the shaded portion of the pole (φ_{sp}) lags the flux (φ_m') in the unshaded portion of the pole resulting in a net torque, which causes the rotor to rotate from the unshaded to the shaded portion of the pole. The motor thus has a definite direction of rotation, which cannot be reversed. Atypical torque speed characteristic are shows in Figure 15 (iv).

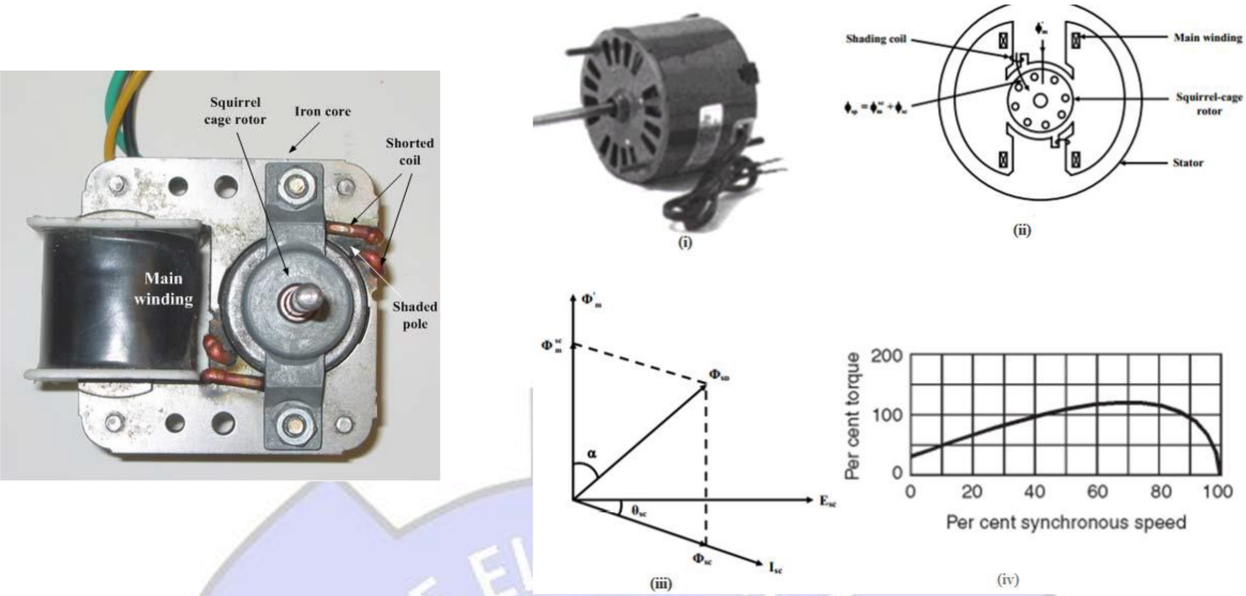


Fig. 15 Shaded-pole induction motors.

7.4.1 Operation

The operation of the motor can be understood by referring to Figure (16) which shows one pole of the motor with a shading coil.

(i) During the portion OA of the alternating-current cycle [See Figure (16)], the flux begins to increase and an EMF. is induced in the shading coil. The resulting current in the shading coil will be in such a direction so as to oppose the change in flux. Thus the flux in the shaded portion of the pole is weakened while that in the unshaded portion is strengthened as shown in Figure (16 (ii)).

(ii) During the portion AB of the alternating-current cycle, the flux has reached almost maximum value and is not changing. Consequently, the flux distribution across the pole is uniform [See Figure (16 (iii))] since no current is flowing in the shading coil. As the flux decreases (portion BC of the alternating current cycle), current is induced in the shading coil so as to oppose the decrease in current. Thus the flux in the shaded portion of the pole is strengthened while that in the unshaded portion is weakened as shown in Figure (16 (iv)).

(iii) The effect of the shading coil is to cause the field flux to shift across the pole face from the unshaded to the shaded portion. This shifting flux is like a rotating weak field moving in the direction from unshaded portion to the shaded portion of the pole.

(iv) The rotor is of the squirrel-cage type and is under the influence of this moving field. Consequently, a small starting torque is developed. As soon as this torque starts to revolve the rotor, additional torque is produced by single-phase induction-motor action. The motor accelerates to a speed slightly below the synchronous speed and runs as a single-phase induction motor.

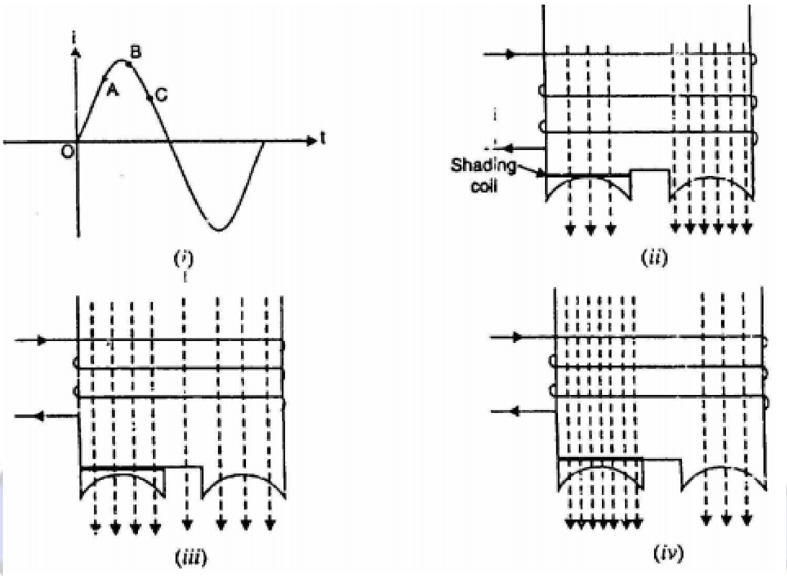


Fig. 16 one pole of the motor with a shading coil.

7.4.2 Characteristics

(i) The salient features of this motor are extremely simple construction and absence of centrifugal switch.

(ii) The motor efficiency is poor, but it is cheap.

(iii) Since starting torque, efficiency and power factor are very low, these motors are only suitable for low power applications e.g., to drive:

(a) Small fans (6) toys (c) hair driers (d) desk fans etc.

8 Equivalent circuit of single phase induction motor

When the stator of single phase induction motor is connected to single – phase supply, the stator current produces a pulsating flux. According to the double – revolving field theory, the pulsating air – gap flux in the motor at standstill can be resolved into two equal and opposite fluxes with the motor. Since the magnitude of each rotating flux is one – half of the alternating flux, it is convenient to assume that the two rotating fluxes are acting on two separate rotors. Thus, a single – phase induction motor may be considered as consisting of two motors having a common stator winding and two imaginary rotors, which rotate in opposite directions.

8.1 At standstill condition

The equivalent circuit of single – phase induction motor is shown in Figure 17

Where :

R1 = resistance of stator winding

X 1 = leakage reactance of stator winding

Xm = total magnetizing reactance

R2 = resistance of rotor referred to the stator

X2 = leakage reactance of rotor referred to the stator

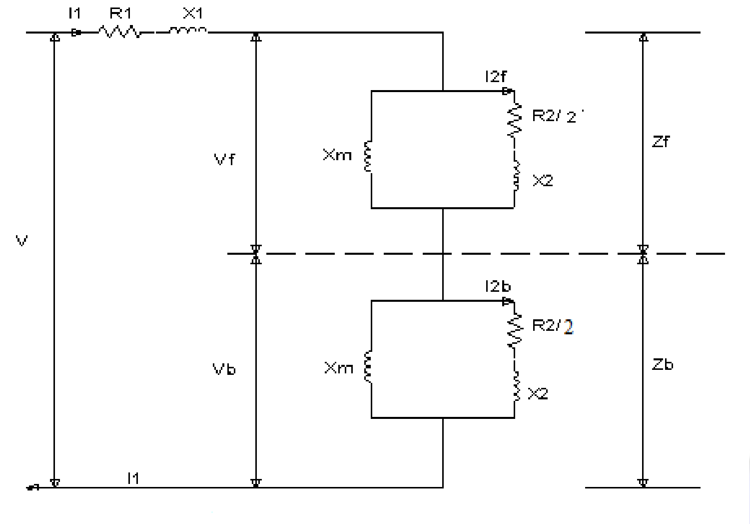


Fig. 17 Equivalent Circuit of Single Phase Induction Motor at Standstill.

At standstill,

φf = φb Therefore, Vf = Vb

Vb = I1Zf

Vb = I1Zb

Zf = Zb

Zf= impedance of forward parallel branch

Zb = impedance of backward parallel branch

$$\boldsymbol{I_1} = \frac{\boldsymbol{V}}{\boldsymbol{Z_t}}$$

Where Zt = Z1 + Zf + Zb

Z1= R1+X1

Zf = Zb

$$\mathrm{Zf} = \frac{JXm\,.\,(\frac{\mathrm{R2}}{2} + j\frac{X2}{2})}{\frac{R2}{2} + j(\,Xm + \frac{X2}{2})}$$

The torque of the backward field is in opposite direction to that of the forward field, and therefore the total air – gap power in a single phase induction motor is

$Pg = Pf - Pb$

Where Pf = air – gap power for forward field

$Pf = I2Rf$

Pb= air – gap power for backward field

$Pb= I2Rb$

The torque produced by the forward field is

$$Tf = \frac{Pf}{w} = \frac{Pf}{2\pi n}$$

The torque produced by the backward field

$$Tb = \frac{Pb}{ws} = \frac{Pb}{2\pi n}$$

The resultant electromagnetic or induced torque Tin is the difference between the torque Tf and Tb

$$Tin = Tf - Tb$$

8.2 At running condition

Now consider that the motor is pinning at some speed in the direction of the forward revolving field, the slip being s. The rotor current produced by the forward field will have a frequency sf where f is the stator frequency. Also, the rotor current produced by the backward field will have a frequency of (2 − s)f. Figure 18 shows the equivalent circuit of a single-phase induction motor when the rotor is rotating at slip s. It is clear, from the equivalent circuit that under running conditions, Ef becomes much greater than Eb because the term R'2/2s increases very much as s tends towards zero. Conversely, E^ falls because the term R'2/2(2 − s) decreases since (2 − s) tends toward 2. Consequently, the forward field increases, increasing the driving torque while the backward field decreases reducing the opposing torque.

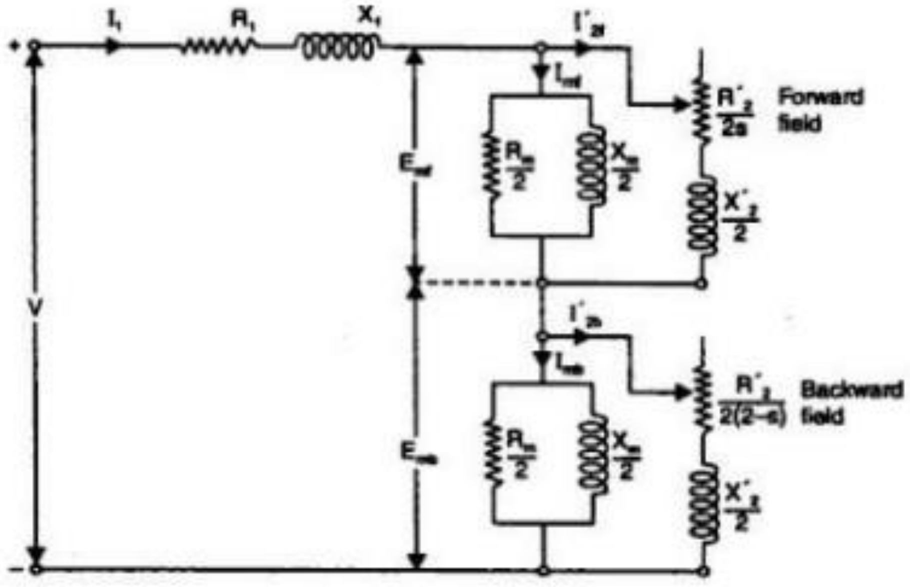


Fig. 18 equivalent circuit of single phase induction motor at operation without core loss.

Zt = Z1 + Zf + Zb

Z1= R1+X1

$$Zf = \frac{JXm\,.\,(\frac{R2}{2S} + j\frac{X2}{2})}{\frac{R2}{2S} + j(\,Xm + \frac{X2}{2})}$$

$$Zb = \frac{JXm\,.\,(\frac{R2}{2(2-S)} + j\frac{X2}{2})}{\frac{R2}{2(2-S)} + j(\,Xm + \frac{X2}{2})}$$

The total copper loss is the sum of rotor copper loss due to the forward field and the rotor copper loss due to the backward field.

Pcu = Pcuf + Pcub
Where
*Pcu= Slip * Pg*
Pcu= S Pgf -+ (2-S) Pgb
The power converted from electrical to mechanical form in a single phase induction motor is given by
Pmech = (1-S)Pg
Shaft output power
Pout =Pmech – friction loss – windage loss

Example 1: A 230 V, 50 Hz, 4 – pole single phase induction motor has the following equivalent circuit impedances:
R1 = 2.2Ω, R2 = 4.5Ω, X1 = 3.1Ω, X2 = 2.6Ω, Xm = 80Ω,
Friction, windage and core loss = 40 W . For a slip of 0.03pu, calculation (a) input current, (b) power factor, (c) developed power, (d) output power, (e) efficiency

Solution:
R2/2S = 4.5/2* 0.03 = 75 Ω
R2/2(2-S) = 4.5/2*(2 -0.03) = 1.142 Ω
X2 /2 = 2.6/2 = 1.3 Ω
Xm/2 = 80/2 = 40 Ω

$$Zf = \frac{JXm\,.\,(\frac{R2}{2S} + j\frac{X2}{2})}{\frac{R2}{2S} + j(\,Xm + \frac{X2}{2})} = 16.37 + j30.98$$

$$Zb = \frac{JXm\,.\,(\frac{R2}{2(2-S)} + j\frac{X2}{2})}{\frac{R2}{2(2-S)} + j(\,Xm + \frac{X2}{2})} = 1.07 + j1.92$$

$Z_1 = R_1 + X_1 = 2.2 + j3.1$

$Zt = Z_1 + Zf + Zb = 19.64 + j\,35.37 = 40.457 < 60.96$

a) Input current

I = V/ Zt = 230 < 0 / 40.457 < 60.96 = 5.685 < -60.69 A

b) Power factor

cos (-60.69) = 0.485 Lag

c) Developed power

Pconv = Pmech = I2 (Rf – Rb) (1-S) = (5.685)2 (16.37 – 1.07) (1 – 0.03)
= 479.65 W

d) Output power

Pout = Pmech - loss = 479.65 - 40 = 439.65 W

Input power = VI cos φ = 230 * 5.685 * 0.485 = 634.9 W

e) Efficiency = Pout / Pin*100% = 0.692 %